GEOMETRIC FORMULAS & THEOREMS

Standards 1, 2, 3

REASONING AND PROOFS

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CONDITIONAL STATEMENTS OR CONDITIONALS:

IF…, THEN …

$$\underbrace{p \longrightarrow q}_{\text{If } p\text{, then } q}$$

Where:

p = hypothesis

q = conclusion

CONVERSE:

IF…, THEN …

$$\underbrace{q \longrightarrow p}_{\text{If } q\text{, then } p}$$

Where:

p = conclusion

q = hypothesis

NEGATION:

The negation of a statement is its denial.

$\sim p$ is "not p" or the negation of p.

INVERSE:

The inverse of a conditional statement is when both the hypothesis and the conclusion are denied.

$$\sim p \longrightarrow \sim q$$

CONTRAPOSITIVE of a conditional statement:

The contrapositive of a conditional statement is the negation of the hypothesis and conclusion of its converse.

IF…, THEN …

$$\underbrace{p \longrightarrow q}_{\text{If } p\text{, then } q}$$

CONVERSE:

IF…, THEN …

$$\underbrace{q \longrightarrow p}_{\text{If } q\text{, then } p}$$

CONTRAPOSITIVE:

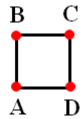
IF…, THEN …

$$\underbrace{\sim q \longrightarrow \sim p}_{\text{If not } q\text{, then not } p}$$

Determine the validity of the conjecture and give a ***counterexample*** should the conjecture be false.

Given:

Points A, B, C, D

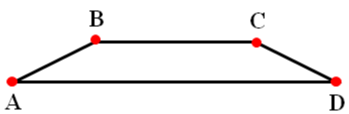


Conjecture:

They only form a square.

False!

Counterexample:



They could form an isosceles trapezoid as well!

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Logical Reasoning

Deductive Reasoning

- Uses a set of rules to prove a statement.

Given:

$4x + 2 = 22$

Prove:

$x = 5$

Proof:

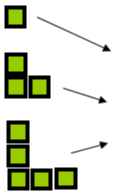
$4x + \cancel{2} = 22$
$\quad -\cancel{2} \quad -2$ ← Subtraction Property of Equality

$\frac{\cancel{4}x}{\cancel{4}} = \frac{20}{4}$ ← Division Property of Equality

$x = 5$ ← Substitution Property of Equality

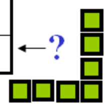
Inductive Reasoning

- Finding a general rule based on observation of data, patterns, and past performance.



Step	Squares
1	1
2	3
3	5
4	7

← ?



Rule: We add 2 squares per step.

Congruence in segments and angles is Reflexive, Symmetric and Transitive:

$\cong$ of segments is *reflexive.*

$\overline{LM} \cong \overline{LM}$

$\cong$ of segments is *symmetric.*

$\overline{KL} \cong \overline{LM} \qquad \overline{LM} \cong \overline{KL}$

$\cong$ of segments is *transitive.*

$\overline{KL} \cong \overline{LM}$
$\overline{LM} \cong \overline{AB}$
$\overline{KL} \cong \overline{AB}$

$\cong$ of $\angle$s is *reflexive*

$\angle ECA \cong \angle ECA$

$\cong$ of $\angle$s is *symmetric*

$\angle BCE \cong \angle FGH \qquad \angle FGH \cong \angle BCE$

$\cong$ of $\angle$s is *transitive*

$\angle BCE \cong \angle FGH$
$\angle FGH \cong \angle ECA$
$\angle BCE \cong \angle ECA$

For all segments and angles, their measures comply with these same properties.

Deductive Reasoning (GEOMETRY)

Conjecture - a statement or conditional trying to prove.

Elements to construct proofs:

a) Undefined terms - Terms that are so obvious that don't require to be proven.

Point and line.

b) Definitions - Statements defined using other terms.

Triangle is a 3 sided polygon.

c) Axioms (Postulates) - Statements or properties that don't need to be proven to be used in proofs.

If two planes intersect their intersection is a line.

d) Theorems - Statements or properties that require to be proven to be used in proofs.

If two angles form a linear pair, then they are supplementary angles.

Indirect Proof: A proof in which you assume the opposite of what you want to prove is true until you find a contradiction.

SEGMENT ADDITION POSTULATE

$$\overline{AB} + \overline{BC} = \overline{AC}$$

- If B is between A and C then $\overline{AB} + \overline{BC} = \overline{AC}$.
- If $\overline{AB} + \overline{BC} = \overline{AC}$, then B is between A and C.

} SEGMENT ADDITION POSTULATE.

SYMBOLS TO REMEMBER

Example	Meaning
$\angle X$	Angle X
$m\angle X$	Measure of angle X
$\overline{XM}$	Line Segment XM
XM	Measure of segment XM
$\overleftrightarrow{KL}$	Line KL
$\overrightarrow{KL}$	Ray KL
ΔKML	Triangle KML
▭WXYZ	Rectangle WXYZ
▱WXYZ	Parallelogram WXYZ

Example	Meaning
$\overleftrightarrow{XY} \parallel \overleftrightarrow{RP}$	Line XY parallel to line RP
$\overleftrightarrow{XY} \perp \overleftrightarrow{RP}$	Line XY perpendicular to line RP
$\overline{WX} \cong \overline{YZ}$	Segments WX andYZ are congruent
$\Delta ABC \sim \Delta STU$	Triangle ABC similar to triangle STU
π	≈ 3.1416 or $\approx 22/7$

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ANGLES & ANGLE RELATIONSHIPS

Standards 2, 7, 13

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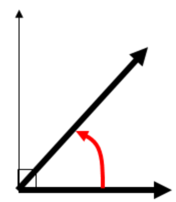
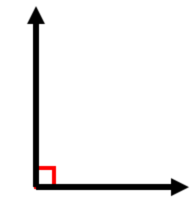
ANGLES CLASSIFICATION



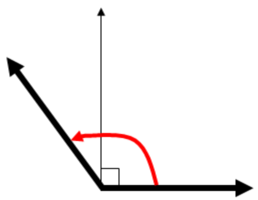
ACUTE ANGLE:

0° < ANGLE < 90°



RIGHT ANGLE:

ANGLE = 90°



OBTUSE ANGLE:

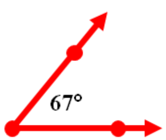
90° < ANGLE < 180°

STRAIGHT ANGLE:

ANGLE = 180°

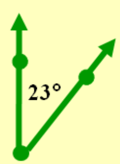
COMPLEMENTARY ANGLES

Find the measure of the COMPLEMENT of the angle below:



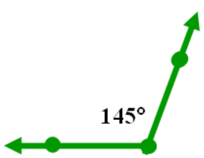
90° – 67° = 23°

Complement



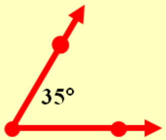
SUPPLEMENTARY ANGLES

Find the measure of the supplement of the angle below:



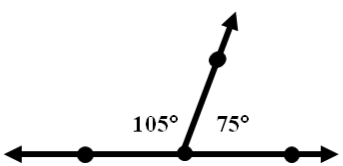
180° –145° = 35°

Supplement:



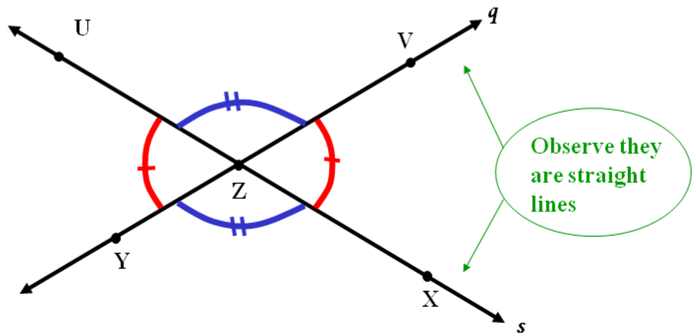
LINEAR PAIR

Let's take a closer look at the angle formation we got before:



- They are supplementary: 105° + 75° = 180°
- They are adjacent: have a common ray.
- *They are LINEAR PAIR: because they lie on a line!*

VERTICAL ANGLES



Means congruence

$\angle UZY \cong \angle VZX \longrightarrow m\angle UZY = m\angle VZX$

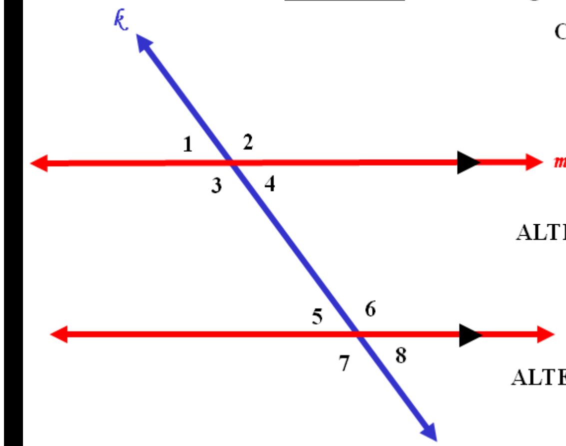
$\angle UZV \cong \angle YZX \longrightarrow m\angle UZV = m\angle YZX$

VERTICAL ANGLES are congruent, that is they have the same measure.

VERTICAL ANGLES are nonadjacent angles formed by two intersecting lines.

ANGLES IN PARALLEL LINES CUT BY A TRANSVERSAL:

If both lines m and ℓ are ***PARALLEL*** the following holds true:



CORRESPONDING Angles are $\cong$:

$\angle 1 \cong \angle 5$
$\angle 3 \cong \angle 7$
$\angle 2 \cong \angle 6$
$\angle 4 \cong \angle 8$

ALTERNATE INTERIOR Angles are $\cong$:

$\angle 3 \cong \angle 6$
$\angle 4 \cong \angle 5$

ALTERNATE EXTERIOR Angles $\cong$

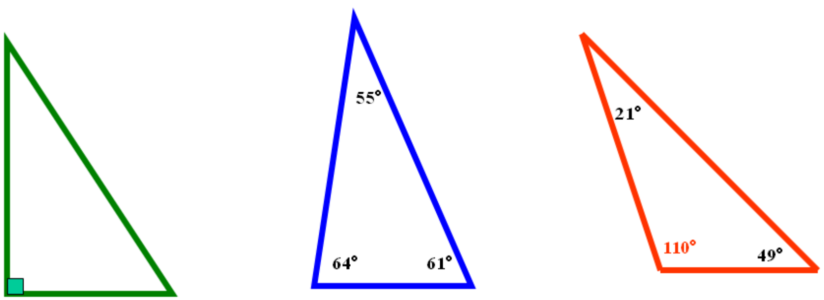
$\angle 1 \cong \angle 8$
$\angle 7 \cong \angle 2$

CONSECUTIVE Interior Angles or SAME SIDE Interior Angles are supplementary:

$m\angle 3 + m\angle 5 = 180°$

$m\angle 4 + m\angle 6 = 180°$

CLASSIFYING TRIANGLES BY **ANGLES**

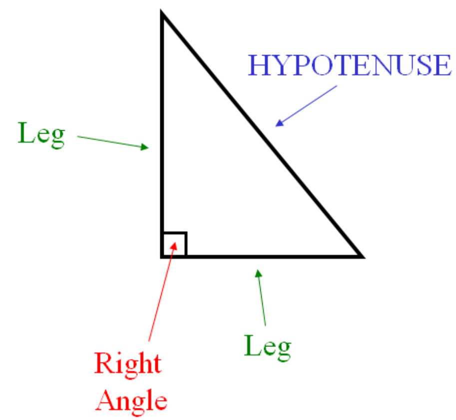


RIGHT TRIANGLE

ACUTE TRIANGLE

OBTUSE TRIANGLE

Parts of a RIGHT TRIANGLE



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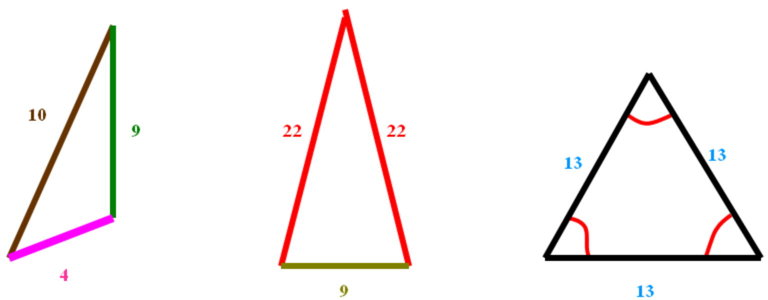
CLASSIFYING TRIANGLES BY **SIDES**



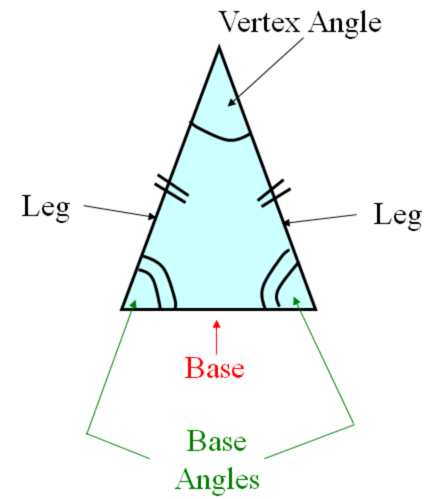
SCALENE TRIANGLE

ISOSCELES TRIANGLE

EQUILATERAL TRIANGLE

Also EQUIANGULAR

Parts of an ISOSCELES TRIANGLE



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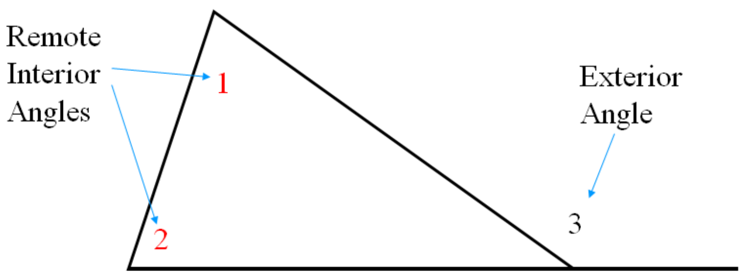
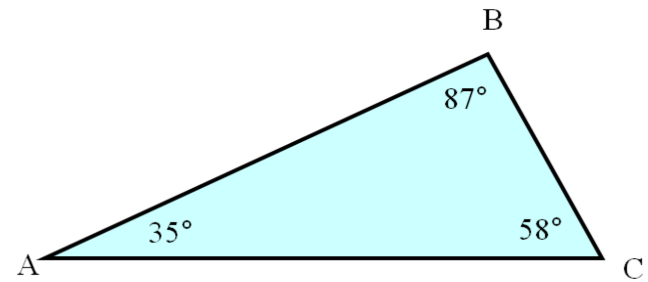
Exterior Angle Theorem



$$m\angle 1 + m\angle 2 = m\angle 3$$

The measure of an exterior angle (angle 3) of a triangle is equal to the sum of the measures of the two remote interior angles (angles 1 and 2).

ANGLE SUM THEOREM:



$$m\angle A + m\angle B + m\angle C = 180°$$

$$35° + 87° + 58° = 180°$$

The sum of the interior angles of a triangle is *always 180°*

CONGRUENT TRIANGLES & TRIANGLE INEQUALITY

Standards 2, 4, 12, 16

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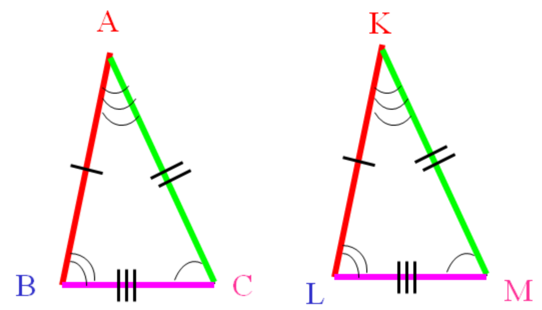
CONGRUENT TRIANGLES

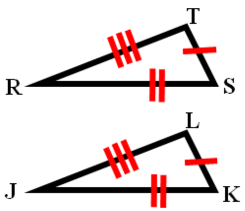


$\Delta ABC \cong \Delta KLM$ by ***CPCTC***

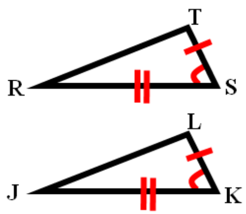
C*orresponding* ***P****arts of* ***C****ongruent* ***T****riangles are* ***C****ongruent*

We had to prove that all six pairs of corresponding parts are congruent!

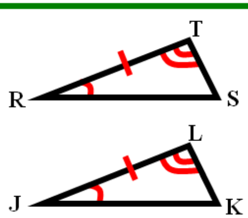
SUMMARY: CONGRUENCE THEOREMS IN TRIANGLES



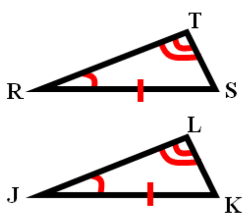
$\Delta RST \cong \Delta JKL$ by ***SSS***



$\Delta RST \cong \Delta JKL$ by ***SAS***

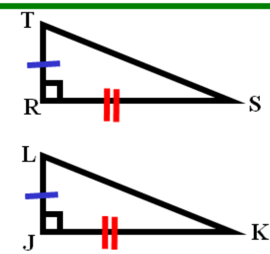


$\Delta RST \cong \Delta JKL$ by ***ASA***

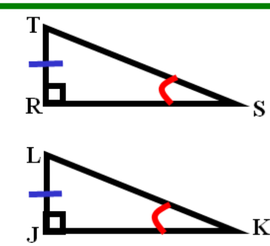


$\Delta RST \cong \Delta JKL$ by ***AAS***

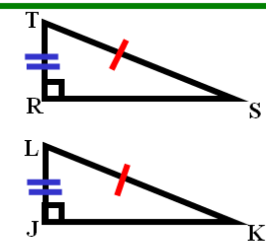
SUMMARY: CONGRUENCE THEOREMS IN RIGHT TRIANGLES



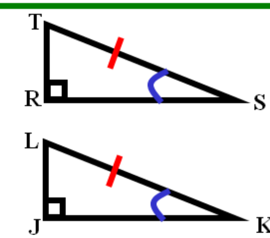
$\Delta RST \cong \Delta JKL$ by ***LL***



$\Delta RST \cong \Delta JKL$ by ***LA***



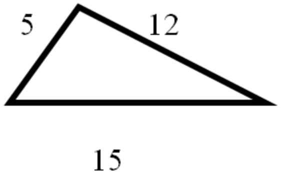
$\Delta RST \cong \Delta JKL$ by ***HL***



$\Delta RST \cong \Delta JKL$ by ***HA***

TRIANGLE INEQUALITY

The sum of the lengths of any two sides of a triangle is greater than the third side.



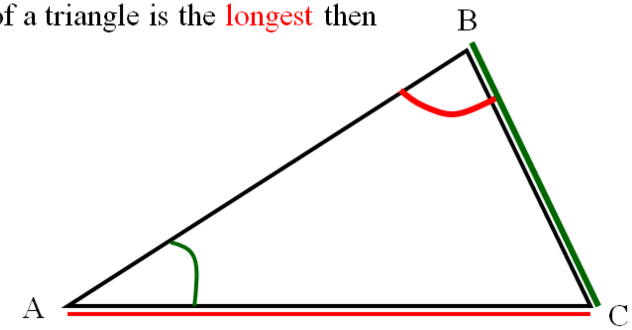
$5+12>15$ or $17>15$

$5+15>12$ or $20>12$

$12+15>5$ or $27>5$

TRIANGLE INEQUALITY ORDERING ANGLES:

If one side of a triangle is the longest then

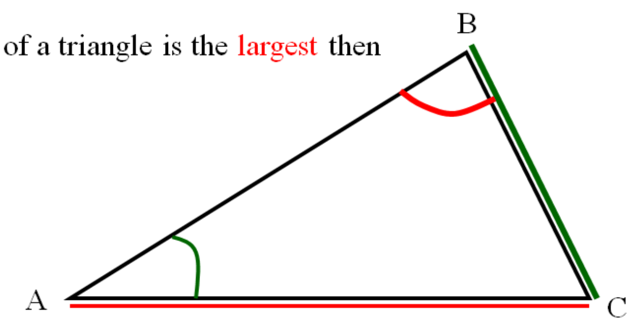


The opposite angle to this side is the largest
And the angle opposite to the shortest side is the smallest

$m\angle B > m\angle C > m\angle A$

TRIANGLE INEQUALITY ORDERING SIDES:

If one angle of a triangle is the largest then



The opposite side to this angle is the longest
And the side opposite to the smallest angle is the shortest

$AC > AB > BC$

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PROPORTIONAL REASONING & SIMILAR TRIANGLES

Standards 2, 3, 4, 5, 12

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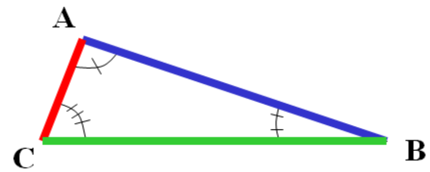
SSS similarity:

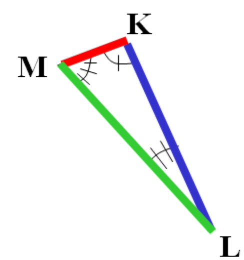
If $\angle A \cong \angle K$ and $\angle B \cong \angle L$ and $\angle C \cong \angle M$ then Triangles are SIMILAR when the corresponding sides are proportional: SSS similarity



$$\frac{AB}{KL}=\frac{CA}{MK}=\frac{BC}{LM}$$

OR

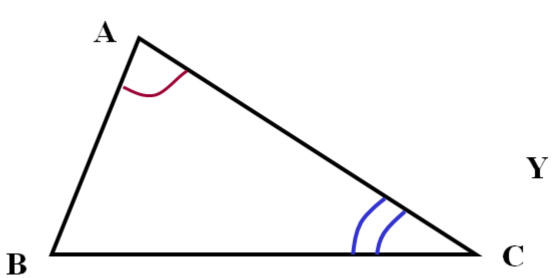
$$\frac{KL}{AB}=\frac{MK}{CA}=\frac{LM}{BC}$$

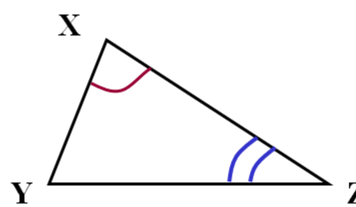


$\Delta CAB \sim \Delta MKL$

Both triangles are similar ($\sim$)

AA similarity:

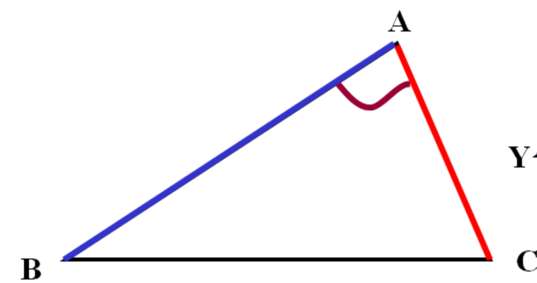


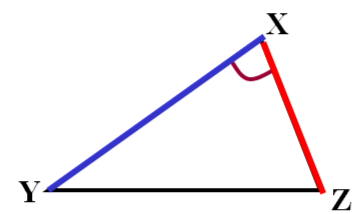


if $\angle A \cong \angle X$ and $\angle C \cong \angle Z$ then $\Delta ABC \sim \Delta XYZ$

By AA similarity

SAS similarity:

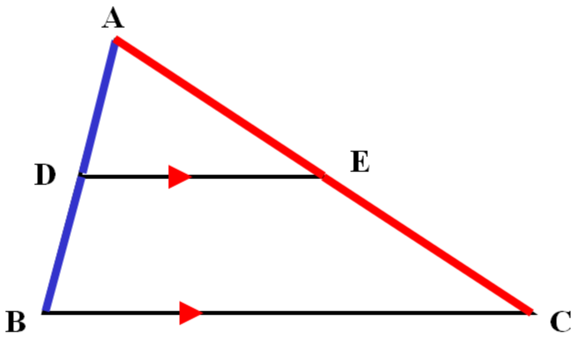




if $\frac{AB}{XY}=\frac{AC}{XZ}$ and $\angle A \cong \angle X$ then $\Delta ABC \sim \Delta XYZ$

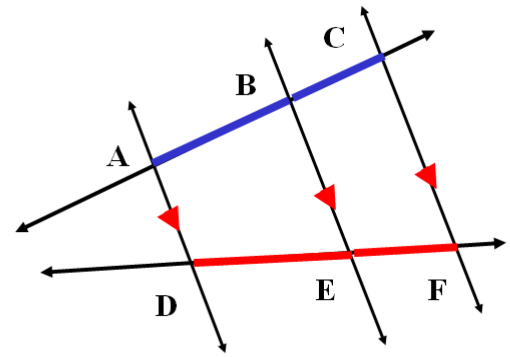
By SAS similarity

SEGMENT PARALLEL TO THE THIRD SIDE IN A TRIANGLE:



If $\frac{DB}{AD} = \frac{EC}{AE}$ then $\overline{DE} \parallel \overline{BC}$

PROPORTIONALITY IN TRANSVERSAL S SEGMENTS CUT BY PARALLEL LINES.



$\frac{AB}{BC} = \frac{DE}{EF}$

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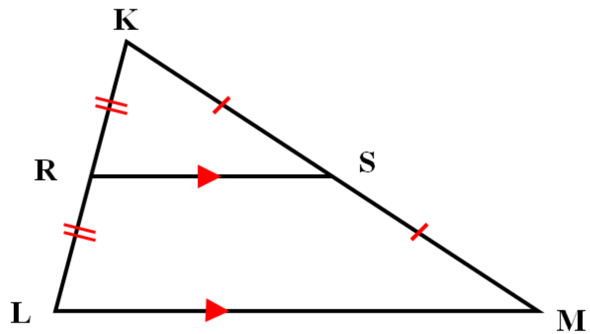
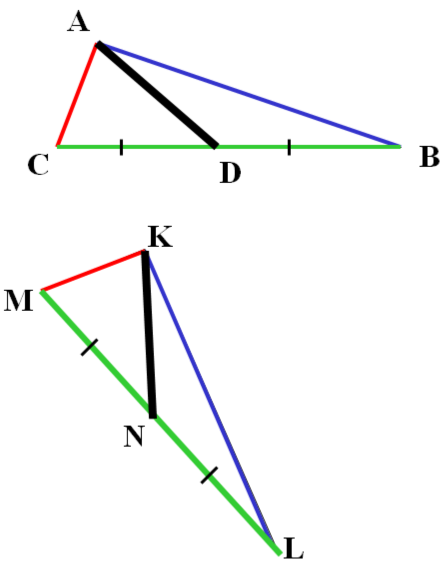
PROPORTIONALITY IN SEGMENTS FORMED BY A SEGMENT JOINING THE MIDPOINT OF TWO SIDES OF A TRIANGLE.



If $\overline{KR} \cong \overline{RL}$ and $\overline{KS} \cong \overline{SM}$ then $\overline{RS} \parallel \overline{LM}$ and $RS = \frac{1}{2} LM$

IN SIMLAR TRIANGLES MEDIANS ARE PROPORTIONAL TO SIDES:

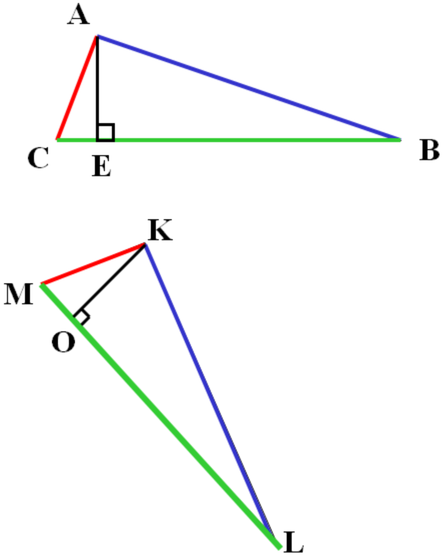


$\frac{AB}{KL} = \frac{CA}{MK} = \frac{BC}{LM} = \frac{AD}{KN}$

OR

$\frac{KL}{AB} = \frac{MK}{CA} = \frac{LM}{BC} = \frac{KN}{AD}$

IN SIMILAR TRIANGLES ALTITUDES ARE PROPORTIONAL TO SIDES:

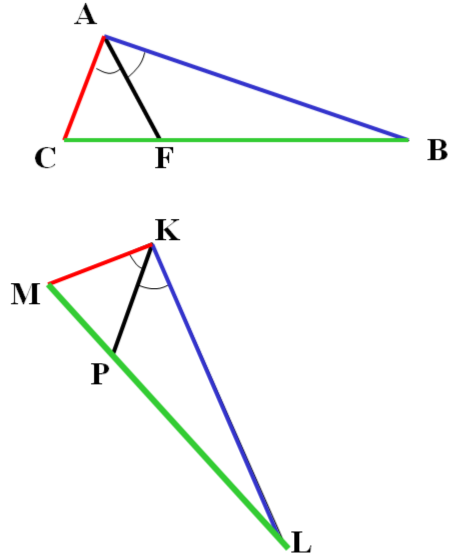


$$\frac{AB}{KL}=\frac{CA}{MK}=\frac{BC}{LM}=\frac{AE}{KO}$$

OR

$$\frac{KL}{AB}=\frac{MK}{CA}=\frac{LM}{BC}=\frac{KO}{AE}$$

IN SIMILAR TRIANGLES ANGLE BISECTORS ARE PROPORTIONAL TO SIDES:

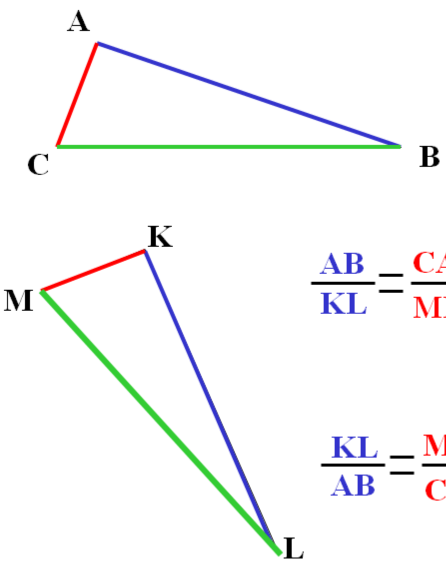


$$\frac{AB}{KL}=\frac{CA}{MK}=\frac{BC}{LM}=\frac{AF}{KP}$$

OR

$$\frac{KL}{AB}=\frac{MK}{CA}=\frac{LM}{BC}=\frac{KP}{AF}$$

IN SIMILAR TRIANGLES PERIMETERS PROPORTIONAL TO SIDES:

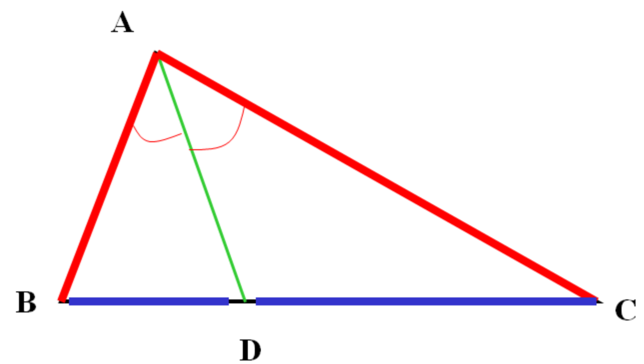


$$\frac{AB}{KL}=\frac{CA}{MK}=\frac{BC}{LM}=\frac{AB+CA+BC}{KL+MK+LM}$$

OR

$$\frac{KL}{AB}=\frac{MK}{CA}=\frac{LM}{BC}=\frac{KL+MK+LM}{AB+CA+BC}$$

SEGMENTS FORMED BY ANGLE BISECTOR PROPORTIONAL TO THE OTHER TWO SIDES.



$$\frac{BD}{DC}=\frac{AB}{AC}$$

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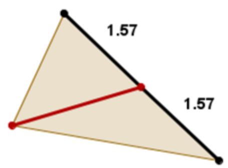
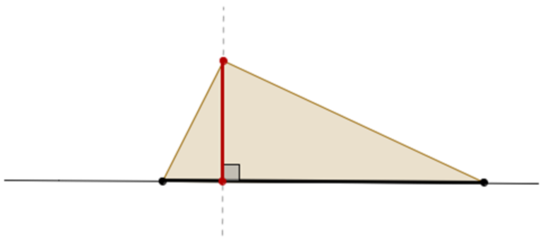
SPECIAL SEGMENTS IN TRIANGLES:

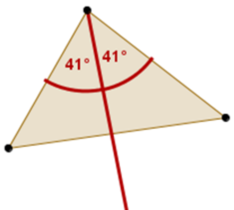
MEDIAN: Segment from vertex to midpoint at the opposite side.



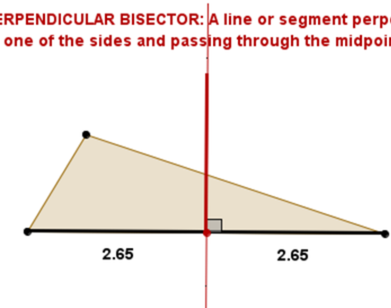
ALTITUDE: Segment from vertex to line containing the opposite side, and forming a right angle with the line.



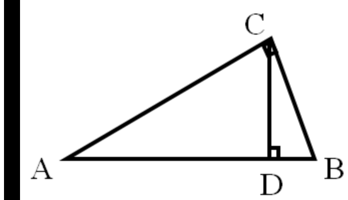
ANGLE BISECTOR: Bisects (divides) the angle into two congruent angles. Angle bisector should be inside the triangle.



PERPENDICULAR BISECTOR: A line or segment perpendicular to one of the sides and passing through the midpoint.



SIMILARITY IN THE TRIANGLES FORMED BY AN ALTITUDE FROM THE RIGHT ANGLE TO THE HYPOTHENUSE:



$\Delta ACB \sim \Delta ADC$

$$\frac{AC}{AD} = \frac{CB}{DC}$$

$$\frac{AC}{AD} = \frac{AB}{AC}$$

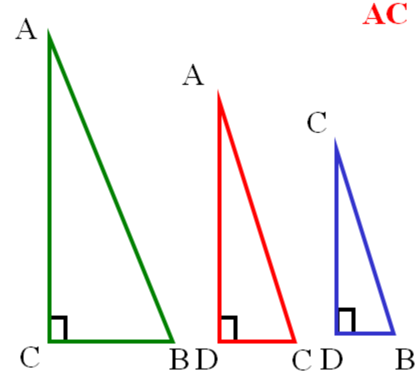
$$\frac{AB}{AC} = \frac{CB}{DC}$$

$\Delta ADC \sim \Delta CDB$

$$\boxed{\frac{AD}{CD} = \frac{CD}{DB}}$$

$$\frac{AD}{CD} = \frac{AC}{CB}$$

$$\frac{AC}{CB} = \frac{DC}{DB}$$



$\Delta ACB \sim \Delta CDB$

$$\frac{AC}{CD} = \frac{CB}{DB}$$

$$\frac{AC}{CD} = \frac{AB}{CB}$$

$$\boxed{\frac{AB}{CB} = \frac{CB}{DB}}$$

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RIGHT TRIANGLES

Standards 12, 14, 15, 16, 18, 19, 20

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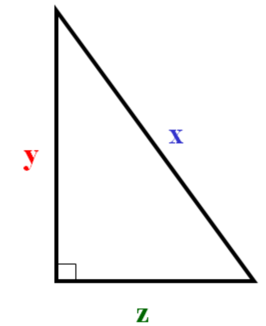
PYTHAGOREAN THEOREM:

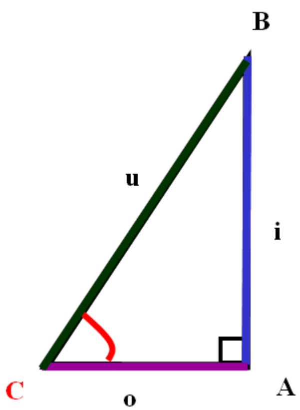


$$y^2 + z^2 = x^2$$

The square of the hypotenuse is equal to the sum of the square of the legs.

The Pythagorean Theorem applies ONLY to RIGHT TRIANGLES!

RIGHT TRIANGLE TRIGONOMETRY:



SINE

$$\text{Sin } C = \frac{\text{Opposite side}}{\text{Hypotenuse}}$$

$$\text{Sin } C = \frac{i}{u}$$

COSINE

$$\text{Cos } C = \frac{\text{Adjacent side}}{\text{Hypotenuse}}$$

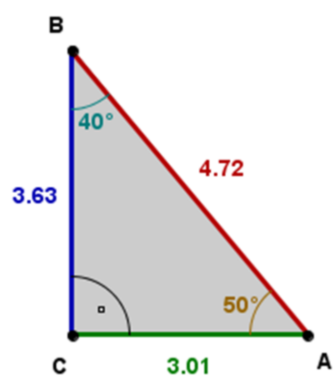
$$\text{Cos } C = \frac{o}{u}$$

TANGENT

$$\text{Tan } C = \frac{\text{Opposite side}}{\text{Adjacent side}}$$

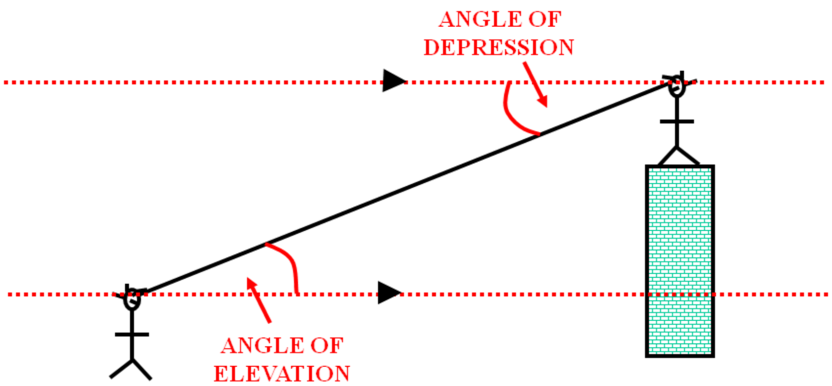
$$\text{Tan } C = \frac{i}{o}$$

RIGHT TRIANGLE TRIGONOMETRY:



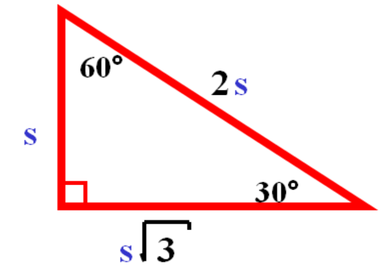
RATIOS	ANGLE
$\text{Sin}(50°) = \frac{3.63}{4.72}$ Sin A = 0.77	arc Sin(0.77) = 50°
$\text{Cos}(50°) = \frac{3.01}{4.72}$ Cos A = 0.64	arc Cos(0.64) = 50°
$\text{Tan}(50°) = \frac{3.63}{3.01}$ Tan A = 1.21	arc Tan(1.21) = 50°

ANGLE OF DEPRESSION AND ANGLE OF ELEVATION



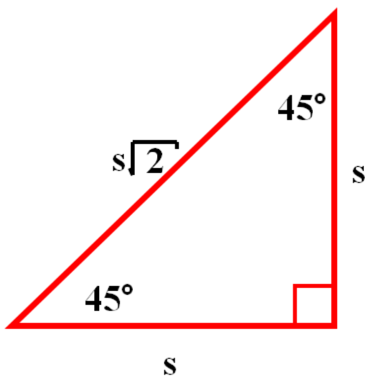
Angles of Depression and Elevation are Alternate Interior Angles and they are congruent.

SPECIAL RIGHT TRIANGLES 30°-60°-90°:



In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

SPECIAL RIGHT TRIANGLES 45°-45°-90°:



In a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times as long as a leg.

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POLYGONS: Areas

Standards 1, 7, 8, 10, 11, 12, 13, 16

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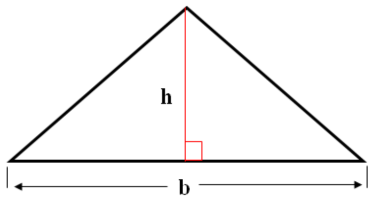
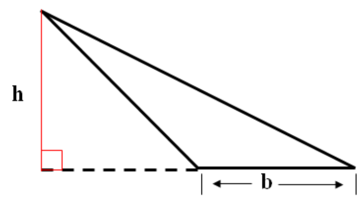
Interior angle sum theorem

If a convex polygon has n sides and S is the sum of the measures of the interior angles, then:

$S = 180°(n-2)$

AREA OF A TRIANGLE



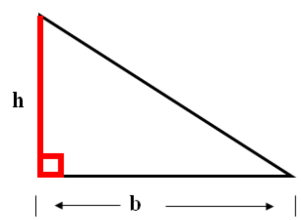


$A = \frac{1}{2}bh$

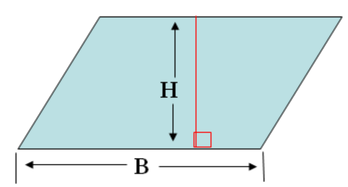
where:

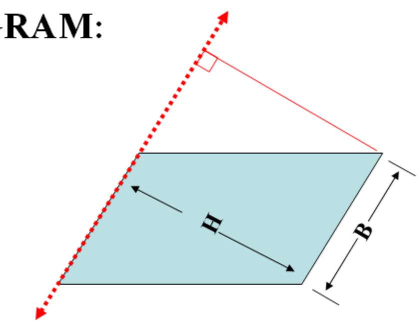
b= base

h= height



AREA OF A PARALLELOGRAM:





$A = B \cdot H$

where:

A= area

B= base

H= height

Could you solve for B?

$\frac{A}{H} = \frac{B \cdot \cancel{H}}{\cancel{H}}$

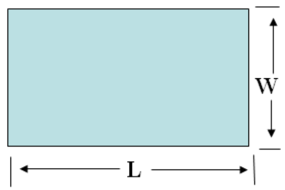
$B = \frac{A}{H}$

Could you solve for H?

$\frac{A}{B} = \frac{\cancel{B} \cdot H}{\cancel{B}}$

$H = \frac{A}{B}$

AREA OF A RECTANGLE:



$A = L \bullet W$

where:

A= area

W= width

L= length

Could you solve for W?

$$\frac{A}{L} = \frac{\cancel{L} \bullet W}{\cancel{L}}$$

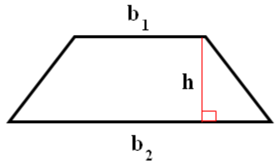
$$W = \frac{A}{L}$$

Could you solve for L?

$$\frac{A}{W} = \frac{L \bullet \cancel{W}}{\cancel{W}}$$

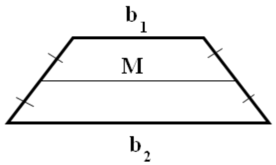
$$L = \frac{A}{W}$$

AREA OF A TRAPEZOID:



Area of Trapezoid = $\frac{1}{2} h \left(b_1 + b_2 \right)$

OR



Area of Trapezoid = $h M$

$$M = \frac{b_1 + b_2}{2}$$

$$M = \frac{1}{2}\left(b_1 + b_2 \right)$$

where:

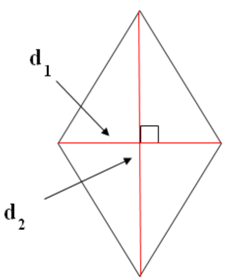
M is the median

b_1 and b_2 are bases of trapezoid

h is the height

Standards 7, 8, 10

AREA OF A RHOMBUS:



$$A = \frac{1}{2} d_1 d_2$$

where:

d_1 = small diagonal

d_2 =large diagonal

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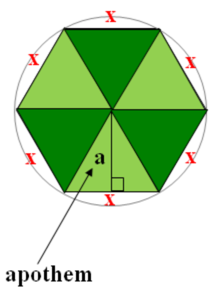
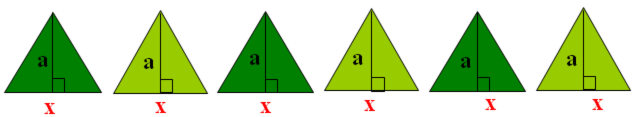
AREA OF A REGULAR POLYGON:

Calculating the area of a regular polygon with side x:

The total Area of the polygon is:





$$A = \frac{1}{2}xa + \frac{1}{2}xa + \frac{1}{2}xa + \frac{1}{2}xa + \frac{1}{2}xa + \frac{1}{2}xa$$

$$A = \frac{1}{2}a\left(x + x + x + x + x + x \right)$$

The perimeter of the polygon is:

$$P = x + x + x + x + x + x$$

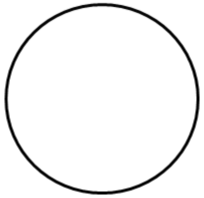
Substituting in Area Formula:

$$A = \frac{1}{2}aP \quad \text{or} \quad A = \frac{1}{2}Pa$$

Area of a Regular Polygon:

If a regular polygon has an *area* of A square units, a *perimeter* of P units, and an *apothem* of a units, then: $A = \frac{1}{2}Pa$

AREA AND PERIMETER OF A CIRCLE:



Perimeter or circumference of the circle.

$C=2\pi r$ or $C=\pi D$

Area of the circle.

$A=\pi r^2$ or $A=\pi\left(\frac{D}{2}\right)^2$

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SOLIDS:
Surface Area & Volume

Standards 1, 8, 9, 11

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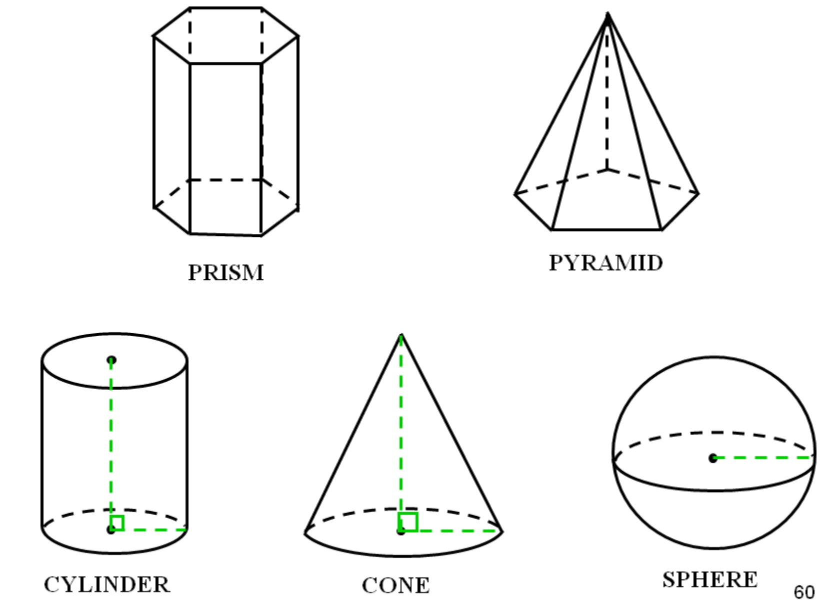
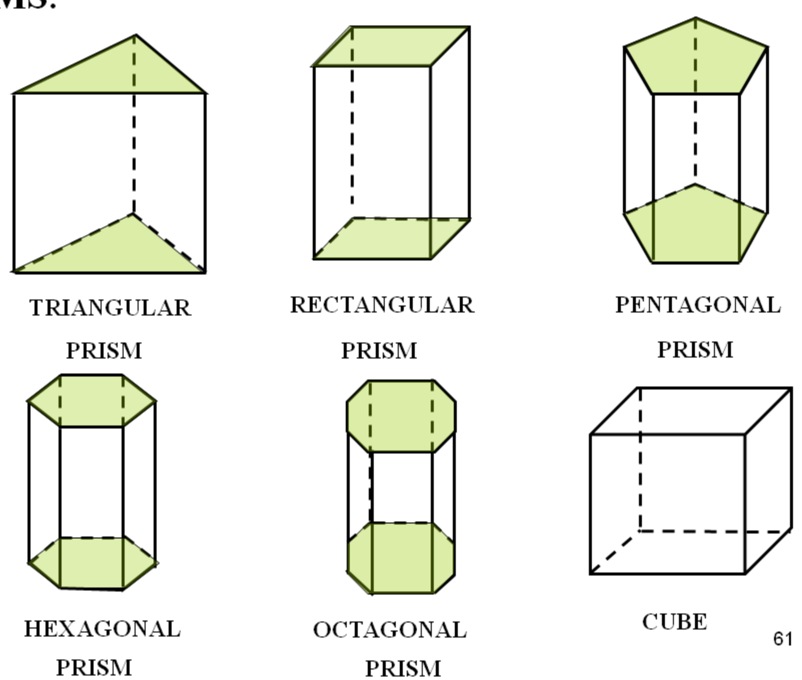
SOLIDS

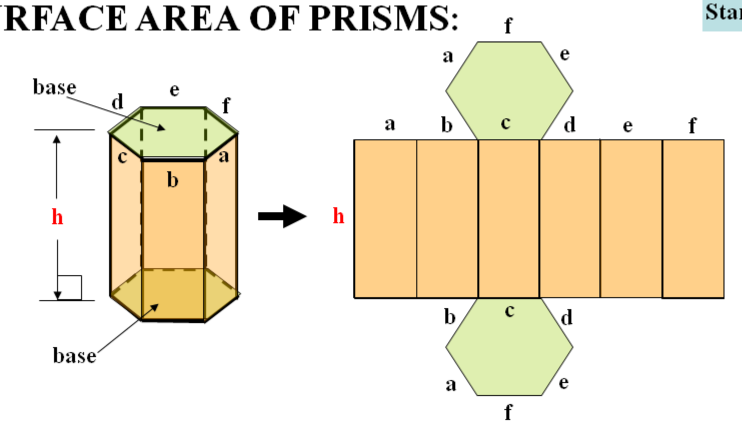


PRIMS:



SURFACE AREA OF PRISMS:

Standards 8, 10, 11



Lateral Area:

$L = ah + bh + ch + dh + eh + fh$

$L = (a + b + c + d + e + f)h$

Perimeter of base:

$P = a + b + c + d + e + f$

Substituting

$L = Ph$

Total Area = Lateral Area + 2 Base Area

$T = L + 2B$

$T = Ph + 2B$

Where:

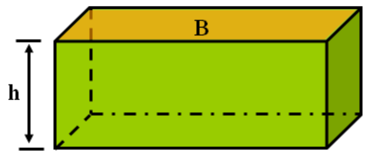
P=perimeter

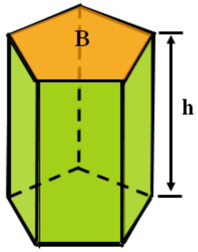
h=height

B= base area

L= lateral area

VOLUME OF A RIGHT PRISM:





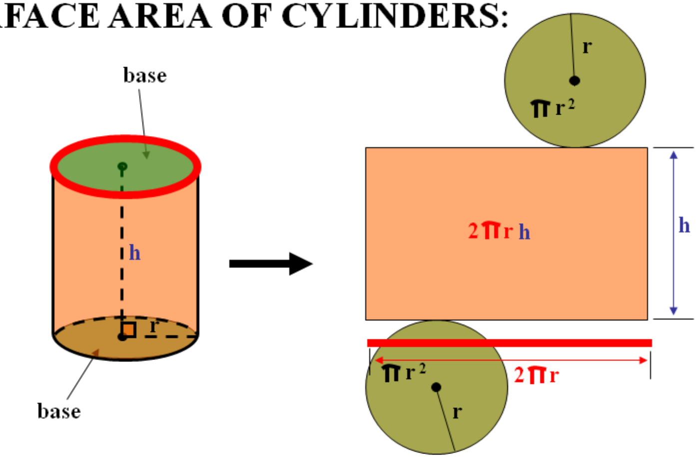
$$\boxed{V = Bh}$$

where:

B= Area of the base

h= height

SURFACE AREA OF CYLINDERS:



Lateral Area:

$L = 2\pi r h$

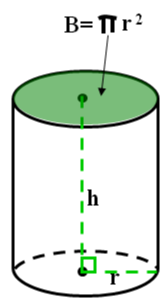
Total Surface Area = Lateral Area + 2(Base Area)

$$\boxed{T = 2\pi r h + 2\pi r^2}$$

h= height

r= radius

VOLUME OF CYLINDERS:



RIGHT CYLINDER

$V = Bh$

$$\boxed{V = \pi r^2 h}$$

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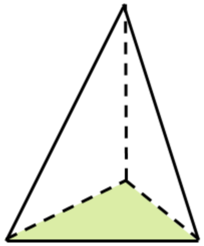
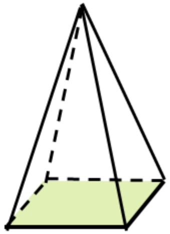
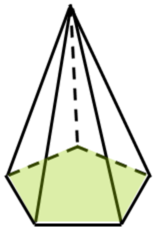
PYRAMIDS:



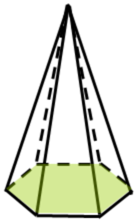
TRIANGULAR PYRAMID



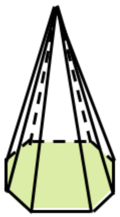
RECTANGULAR PYRAMID



PENTAGONAL PYRAMID

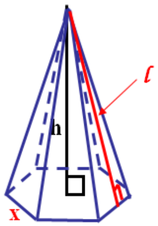


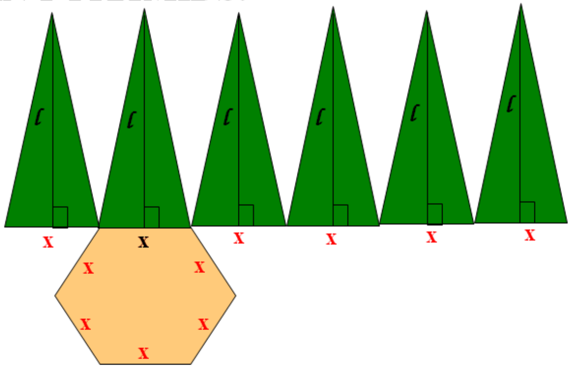
HEXAGONAL PYRAMID



OCTAGONAL PYRAMID

SURFACE AREA IN PYRAMIDS:





Calculating Lateral Area:

$$L = \frac{1}{2}x\ell + \frac{1}{2}x\ell + \frac{1}{2}x\ell + \frac{1}{2}x\ell + \frac{1}{2}x\ell + \frac{1}{2}x\ell$$

$$L = \frac{1}{2}\ell\left(x + x + x + x + x + x\right)$$

The perimeter of the BASE is:

$$P = x + x + x + x + x + x$$

TOTAL SURFACE AREA:

$$\boxed{T = \frac{1}{2}P\ell + B}$$

LATERAL AREA IS:

$$L = \frac{1}{2}P\ell$$

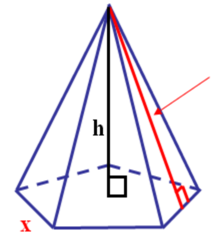
P= perimeter of base

B= Area of base polygon

ℓ= slant height

h= height

VOLUME OF A PYRAMID:



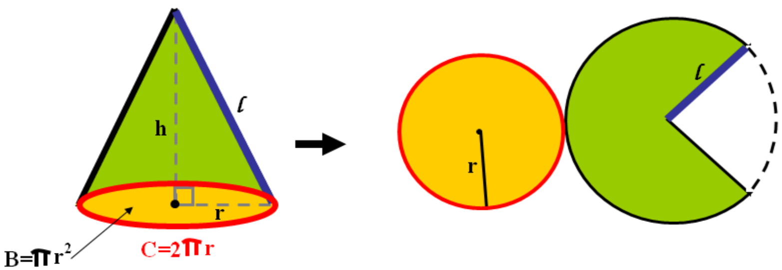
$$\boxed{V = \frac{1}{3}Bh}$$

where:

B= Area of the base

h= height

SURFACE AREA OF A RIGHT CIRCULAR CONE:



TOTAL SURFACE AREA:

T = area of sector + area of cone's base

T = L + B

$$\boxed{T = \pi r\ell + \pi r^2}$$

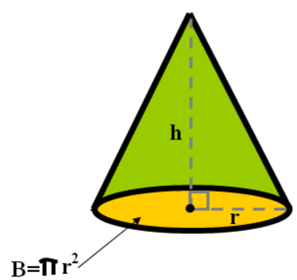
h= height

r = radius

ℓ= slant height

L=area of sector = $\boxed{\pi r \ell}$ ← Lateral Area

VOLUME OF A RIGHT CIRCULAR CONE:



$$V = \frac{1}{3}Bh$$

$$\boxed{V = \frac{1}{3}\pi r^2 h}$$

h= height

r = radius

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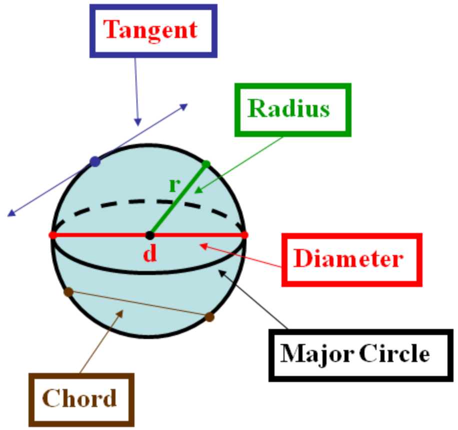
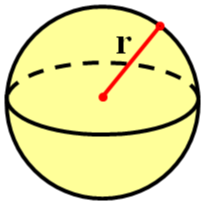
SPHERE FEATURES



SPHERE:



SURFACE AREA OF A SPHERE

$$SA = 4\pi r^2$$

VOLUME OF A SPHERE

$$V = \frac{4}{3}\pi r^3$$

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CIRCLES: ARCS, CHORDS, TANGENTS & SECANTS.

Standards 2, 9, 16, 21

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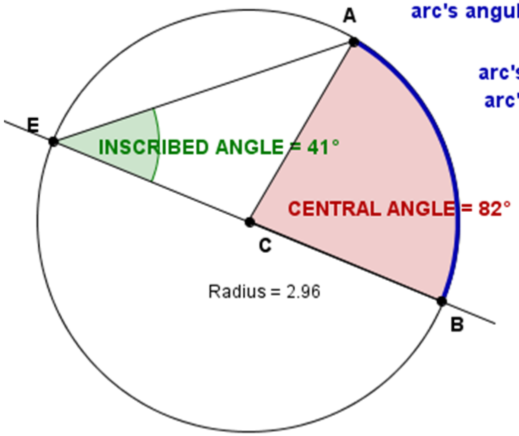
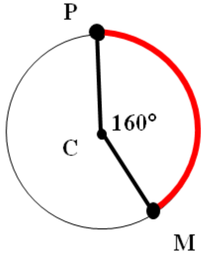
arc's angular measure = Central Angle
arc's angular measure = 82°

arc's length= [(central angle)/360]*2*pi*radius
arc's length = 4.21



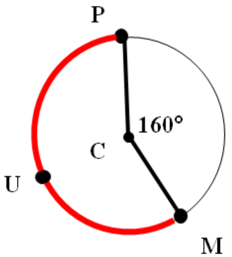
$$\text{ARC'S LENGTH} = \left(\frac{\text{Arc's measure}}{360°}\right)\left(\text{Circumference}\right)$$

SEMICIRCLE, MINOR AND MAJOR ARCS:



The measure of a minor arc is the measure of its central angle.

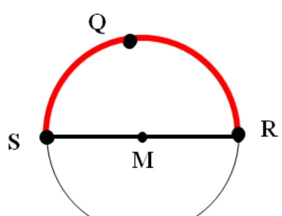
$m\angle PCM = 160° \longrightarrow m\,\widehat{PM} = 160°$



The measure of a major arc is 360° minus the measure of its central angle.

$m\,\widehat{PUM} = 360° - m\angle PCM$

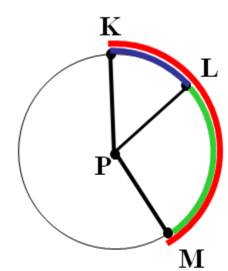
$= 360° - 160° = 200°$



The measure of a SEMICIRCLE is 180°.

$m\,\widehat{SQR} = 180°$

ARC ADDITION POSTULATE:



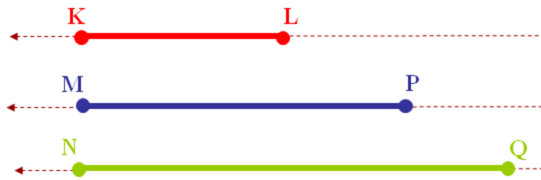
$m\,\widehat{KM} = m\,\widehat{KL} + m\,\widehat{LM}$

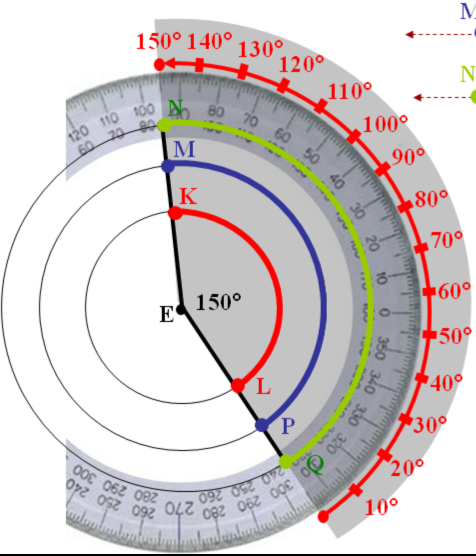
Arc Addition Postulate:

The measure on an arc formed by two adjacent not overlapping arcs is the addition of the measures of the two arcs.

ARC LENGTH VS ANGULAR MEASURE:

Lets extend the arcs over a straight line:





We can see they have different lengths or perimeter:

$KL < MP < NQ$

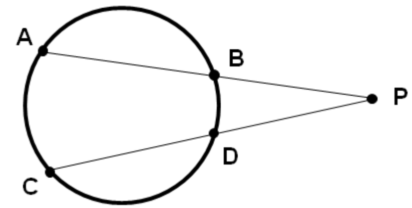
but their central angle and angular measures are the same:

$m\angle KEL = m\,\widehat{KL} = m\,\widehat{MP} = m\,\widehat{NQ} = 150°$

So, we may see that the ARC'S LENGTH is different even though the CENTRAL ANGLE measure and the ANGULAR MEASURE OF THE ARCS are the same!

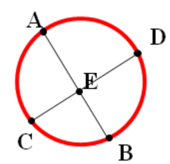
SEGMENTS IN SECANTS INTERSECTING IN AN EXTERIOR POINT OF A CIRCLE:

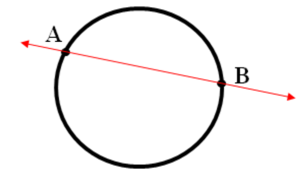
(PC)(PD)=(PA)(PB)



SEGMENTS IN SECANTS INTERSECTING IN AN INTERIOR POINT OF A CIRCLE:

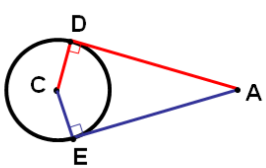
(AE)(EB)=(CE)(ED)





Line q is a SECANT. A secant is a line or segment that intersects a circle in two points.

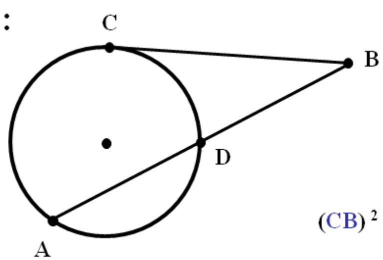
SEGMENTS IN TANGENTS INTERSECTING IN AN EXTERIOR POINT OF A CIRCLE:



(1) Tangents are perpendicular to radii

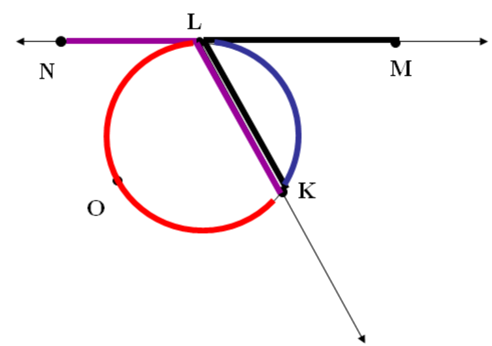
(2) Tangents with a common exterior point are $\cong$

SEGMENTS IN TANGENT AND SECANT INTERSECTING IN AN EXTERIOR POINT OF A CIRCLE:



$(CB)^2 = (AB)(BD)$

ARCS AND ANGLES FORMED BY A SECANT AND A TANGENT INTERSECTING IN THE POINT OF TANGENCY:



$m\angle KLM = \frac{1}{2} m\,\widehat{KL}$

$m\angle KLN = \frac{1}{2} m\,\widehat{KOL}$

If a secant and a tangent intersect at the point of tangency, then the measure of each angle formed is one-half the measure of its intercepted arc.

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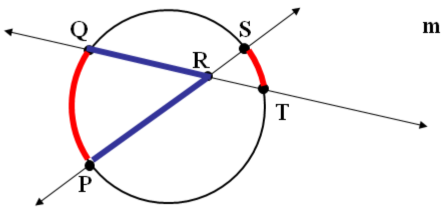
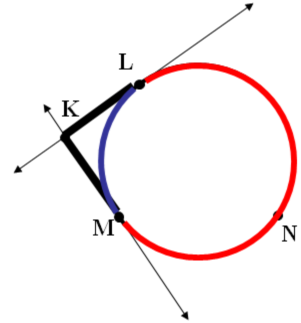
ANGLES AND ARCS FORMED BY SECANTS INTERSECTING IN THE INTERIOR OF THE CIRCLE:



$$m\angle QRP = \frac{1}{2}\left(m\overset{\frown}{ST} + m\overset{\frown}{QP}\right)$$

If two secants intersect in the interior of a circle, then the measure of an angle formed is one-half the sum of the measures of the arcs intercepted by the angle and its vertical angle.

SEGMENTS AND SECANTS INTERSECTING IN AN EXTERIOR POINT THE FORMED ANGLE AND ARCS:

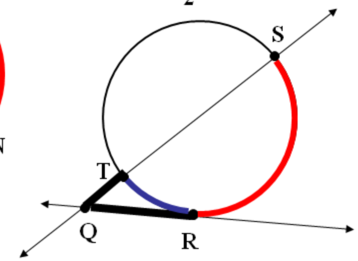


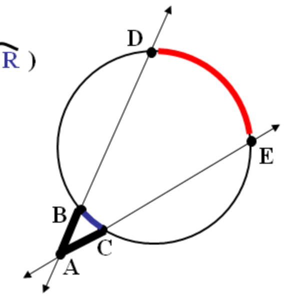
Two tangents

$$m\angle LKM = \frac{1}{2}\left(m\overset{\frown}{LNM} - m\overset{\frown}{LM}\right)$$

Secant and a tangent

$$m\angle TQR = \frac{1}{2}\left(m\overset{\frown}{SR} - m\overset{\frown}{TR}\right)$$





Two secants

$$m\angle BAC = \frac{1}{2}\left(m\overset{\frown}{DE} - m\overset{\frown}{BC}\right)$$

If two secants, a secant and a tangent, or two tangents intersect in the exterior of a circle, then the measure of the angle formed is one-half the possitive difference of the measures of the intercepted arcs.

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COORDINATE GEOMETRY

Slope, Midpoint And Distance For Two Points In The Coordinate Plane.

Standards 1, 7, 2, 22

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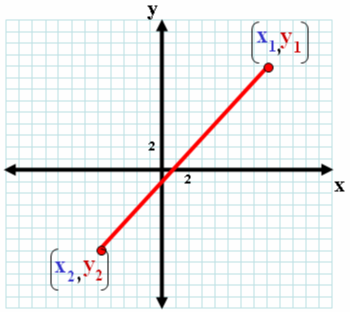
DISTANCE AND SLOPE BETWEEN TWO POINTS IN THE COORDINATE PLANE:

Distance Formula between two points in a plane:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Slope Formula:

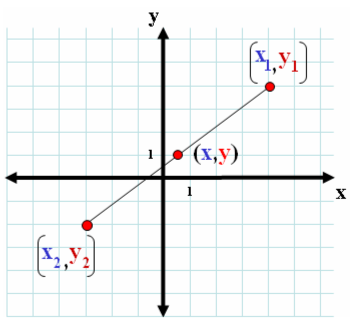
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$$



MIDPOINT OF A LINE SEGMENT:

If a line segment has endpoints at $(x_1 y_1)$ and $(x_2 y_2)$, then the midpoint of the line segment has coordinates:

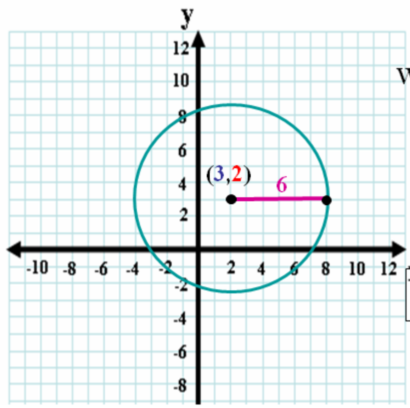
$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$



EQUATION OF A CIRCLE:

The equation of a circle with center at (h,k) and radius r units is

$$\boxed{(x - h)^2 + (y - k)^2 = r^2}$$

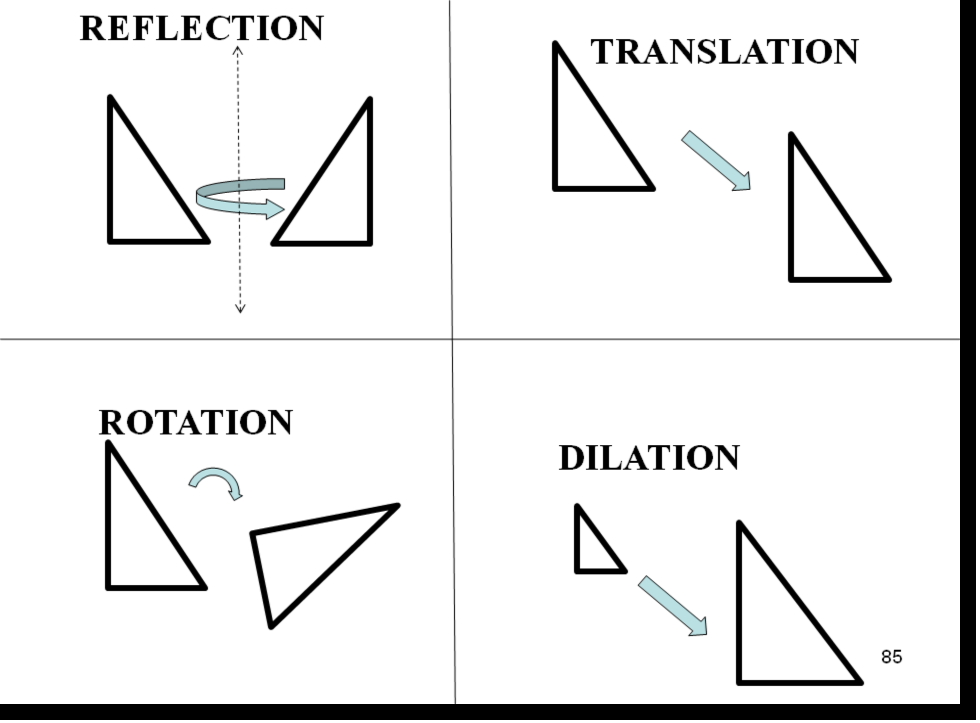


What would be the equation for this circle?

h = 3
k = 2
r = 6

$$(x - 3)^2 + (y - 2)^2 = (6)^2$$

$$\boxed{(x - 3)^2 + (y - 2)^2 = 36}$$



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CONSTRUCTIONS

Segment and Angle Bisectors

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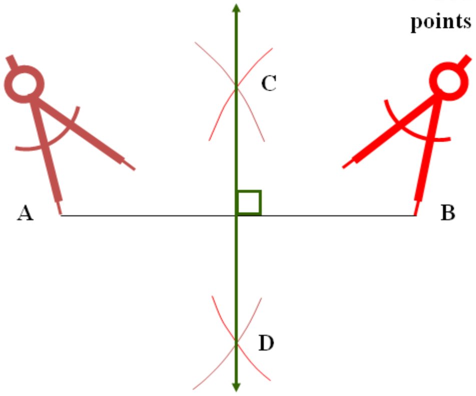
CONSTRUCTING THE PERPENDICULAR BISECTOR OF A LINE.

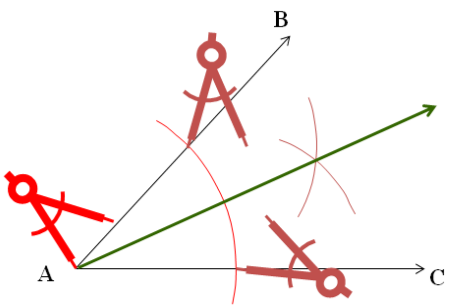
1. Place compass at A, set it more than half distance AB and draw 2 arcs.

2. Place compass at B, ***with same distance*** set and draw 2 arcs to intersect first two.

3. Draw the perpendicular bisector through the points of intersection C and D.



CONSTRUCTING THE ANGLE BISECTOR OF AN ANGLE.



1. Place compass at the angle's vertex, and draw an arc crossing rays AB and AC.

2. Go to each one of the intersections ***using the same distance previously set*** in the compass and draw 2 arcs that intersect.

3. Draw the angle bisector from the vertex to the point were the arcs intersect.

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CALIFORNIA GEOMETRY STANDARDS

CA 1.0 Students demonstrate understanding by identifying and giving examples of undefined terms, axioms, theorems, and inductive and deductive reasoning.

CA 2.0 Students write geometric proofs, including proofs by contradiction.

CA 3.0 Students construct and judge the validity of a logical argument and give counter examples to disprove a statement.

CA 4.0 Students prove basic theorems involving congruence and similarity.

CA 5.0 Students prove that triangles are congruent or similar, and they are able to use the concept of corresponding parts of congruent triangles.

CA 6.0 Students know and are able to use the triangle inequality theorem.

CA 7.0 Students prove and use theorems involving the properties of parallel lines cut by a transversal, the properties of quadrilaterals, and the properties of circles.

CA 8.0 Students know, derive, and solve problems involving the perimeter, circumference, area, volume, lateral area, and surface area of common geometric figures.

CA 9.0 Students compute the volumes and surface areas of prisms, pyramids, cylinders, cones, and spheres; and students commit to memory the formulas for prisms, pyramids, and cylinders.

CA 10.0 Students compute areas of polygons, including rectangles, scalene triangles, equilateral triangles, rhombi, parallelograms, and trapezoids.

CA 11.0 Students determine how changes in dimensions affect the perimeter, area, and volume of common geometric figures and solids.

CA 12.0 Students find and use measures of sides and of interior and exterior angles of triangles and polygons to classify figures and solve problems.

CA 13.0 Students prove relationships between angles in polygons by using properties of complementary, supplementary, vertical, and exterior angles.

CA 14.0 Students prove the Pythagorean theorem.

CA 15.0 Students use the Pythagorean theorem to determine distance and find missing lengths of sides of right triangles.

CA 16.0 Students perform basic constructions with a straightedge and compass, such as angle bisectors, perpendicular bisectors, and the line parallel to a given line through a point of the line.

CA 17.0 Students prove theorems by using (use) coordinate geometry, including the midpoint of a line segment, the distance formula, and various forms of equations of lines and circles.

CA 18.0 Students know the definitions of the basic trigonometric functions defined by the angles of a right triangle. They also know and are able to use elementary relationships between them. For example, tan(x) = sin(x)/cos(x), (sin(x))2 + (cos (x)) 2 = 1.

CA 19.0 Students use trigonometric functions to solve for an unknown length of a side of a right triangle, given an angle and a length of a side.

CA 20.0 Students know and are able to use angle and side relationships in problems with special right triangles, such as 30°, 60°, and 90° triangles and 45°, 45°, and 90° triangles.

CA 21.0 Students prove and solve problems regarding relationships among chords, secants, tangents, inscribed angles, and inscribed and circumscribed polygons of circles.

CA 22.0 Students know the effect of rigid motions on figures in the coordinate plane and space, including rotations, translations, and reflections.

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Fresno, Ca. 2009

Special thanks to Sharon Hart, lead teacher at Roosevelt High School for allowing to use as organization model of this document; the concept and formulas compilation she used to distribute to her students and the math department in preparation for the CST.