Geometric Theorems and Postulates

This version is similar to the previously posted and still available in the website listing a cross reference with Geometric Standars for California. Both the ones before the Common core and the Common Core.

It is a work in progress and might have some errors that will be corrected and we spot them.

Negation:

The negation is the negation of a statement

$\sim p$ ***is "No p" or negating "p"***

An Angle is Rigth ***An Angle is Not Right***

$$p \longrightarrow \sim p$$

An Angle is Not Right ***An Angle is Right***

$$p \longrightarrow \sim p$$

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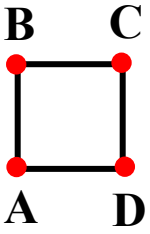
$Counter - Example$:

$Determine\ the\ validity\ of\ the\ CONJECTURE. If\ it\ is\ false\ give\ a\ COUNTER\ EXAMPLE$

Given:

$Points\ A, B, C, D$

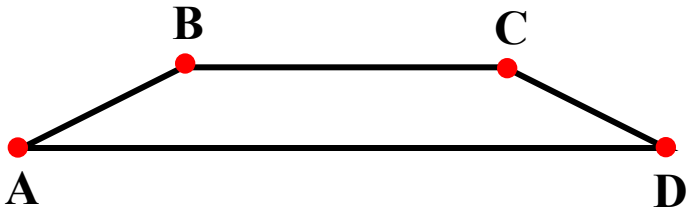


Conjecture:

$It\ is\ only\ possible\ to\ arrange\ points\ in\ a\ square$

False!

$Counter - Example$:



$It\ is\ possible\ to\ arrange\ them\ in\ a\ trapezoid$

Conditional Statements or Propositions:

$$\text{if} \ldots \text{then} \ldots$$
$$p \rightarrow q$$
$$\text{if}\ \{p\}\ \text{then}\ \{q\}$$

Where:
p = hypotesis
q = conclusion

Converse:

$$\text{if} \ldots \text{then} \ldots$$
$$q \rightarrow p$$
$$\text{if}\ \{q\}\ \text{then}\ \{p\}$$

Where:
p = conclusion
q = hypotesis

Inverse:

$$\text{if} \ldots \text{then} \ldots$$
$$\sim p \rightarrow \sim q$$
$$\text{if}\ \{\sim p\}\ \text{then}\ \{\sim q\}$$

Where:
p = hypotesis
q = conclusion

Contrapositive:

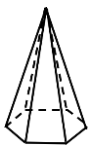
$$\text{if} \ldots \text{then} \ldots$$
$$\sim q \rightarrow \sim p$$
$$\text{if}\ \{\sim q\}\ \text{then}\ \{\sim p\}$$

Where:
p = conclusipn
q = hypotesis
$\sim$ = negation

Law of Detachment:

If $\{p \rightarrow q$ *is a true statement and* p *is true*$\}$ *then* $\{q$ *is true.*$\}$

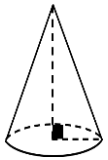
General $\longrightarrow$ ***Particular***



$$If\ \left\{\boldsymbol{S.A.} = \frac{\mathbf{1}}{\mathbf{2}} \cdot \boldsymbol{P} \cdot \boldsymbol{l} + \frac{\mathbf{1}}{\mathbf{2}} \cdot \boldsymbol{P} \cdot \boldsymbol{a}\right\}$$

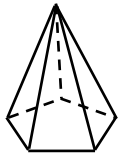
$$then\ \left\{\boldsymbol{S.A.} = \frac{\mathbf{1}}{\mathbf{2}} \cdot \mathbf{36} \cdot \mathbf{26.5} + \frac{\mathbf{1}}{\mathbf{2}} \cdot \mathbf{36} \cdot \mathbf{3} \cdot \sqrt{\mathbf{3}} \rightarrow S.A. = 15 \cdot \pi \approx \mathbf{810}\right\}$$

$$with\ P = 36\ and\ l = 26.5\ and\ a = 3 \cdot \sqrt{3}$$



$$If\ \{\boldsymbol{S.A.} = \boldsymbol{\pi} \cdot \boldsymbol{r}^{\mathbf{2}} + \boldsymbol{\pi} \cdot \boldsymbol{r} \cdot \boldsymbol{l}\}$$

$$then\ \{\boldsymbol{S.A.} = \boldsymbol{\pi} \cdot (\mathbf{3})^{\mathbf{2}} + \boldsymbol{\pi} \cdot \mathbf{3} \cdot \mathbf{2} \rightarrow S.A. = \mathbf{15} \cdot \boldsymbol{\pi}\}\ with\ r = 3\ and\ l = 2$$



$$If\ \left\{\boldsymbol{V} = \frac{\mathbf{1}}{\mathbf{3}} \cdot \boldsymbol{B} \cdot \boldsymbol{h}\right\}\ then\ \left\{\boldsymbol{V} = \frac{\mathbf{1}}{\mathbf{3}} \cdot \mathbf{2} \cdot \mathbf{3} \rightarrow V = \mathbf{2}\right\}\ with\ B = 2\ and\ h = 3$$

$$If\ \{\boldsymbol{S.A.} = \mathbf{4} \cdot \boldsymbol{\pi} \cdot \boldsymbol{r}^{\mathbf{2}}\}\ then\ \{\boldsymbol{S.A.} = \mathbf{4} \cdot \boldsymbol{\pi} \cdot (\mathbf{3})^{\mathbf{2}} \rightarrow S.A. = \mathbf{36} \cdot \boldsymbol{\pi}\}\ with\ r = 3$$

$$If\ \left\{\boldsymbol{V} = \frac{\mathbf{4}}{\mathbf{3}} \cdot \boldsymbol{\pi} \cdot \boldsymbol{r}^{\mathbf{3}}\right\}\ then\ \left\{\boldsymbol{V} = \frac{\mathbf{4}}{\mathbf{3}} \cdot \boldsymbol{\pi} \cdot (\mathbf{3})^{\mathbf{3}} \rightarrow S.A. = \mathbf{12} \cdot \boldsymbol{\pi}\right\}\ with\ r = 3$$

Law of Sylogism:

$$If\ \{\ p \rightarrow q\ \}\ and\ \{\ q \rightarrow r\ \}\ then\ \{\ p \rightarrow r\ \}$$

$If\ \{$ ▭ $\rightarrow 2\ pairs\ of\ \parallel\ Sides\ \}\ and\ \{\ A\ parallelogram \rightarrow 2\ pairs\ of\ \parallel\ Sides\ \}$

$then\ \{$ ▭ $\rightarrow A\ parallelogram\ \}$

$If\ \{\ \angle ABC\ \rightarrow measures\ 60^o\ \}\ and\ \{\ 0^o < angle < 90^o \rightarrow An\ acute\ \angle\}$

$then\ \{\ \angle ABC \rightarrow Acute\ \angle\ \}$

$If\ \{\ A\ kite \rightarrow 2\ pairs\ of\ consecutive\ sides \cong' s\ \}$

$and\ \{\ ABCD\ \rightarrow 2\ pairs\ of\ consecutive\ sides \cong' s\ \}$

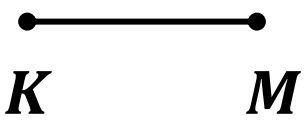
$then\ \{ABCD \rightarrow KITE\}$

$Recall$:

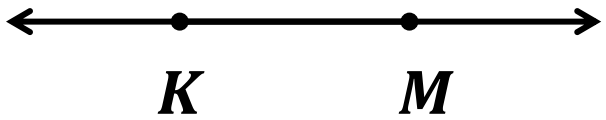
$Transitive\ Porperty\ in\ Algebra$: $if\ \{a = b\}\ and\ \{b = c\}\ then\ \{a = c\}$

Segments, lines and rays

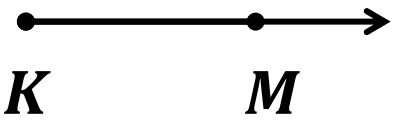
Segment $\overline{KM}$



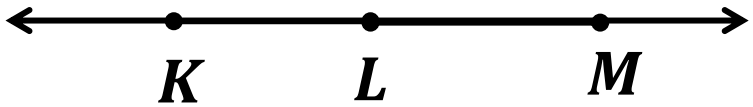
Line $\overleftrightarrow{KM}$



Ray $\overrightarrow{KM}$



Segment Addition Postulate:

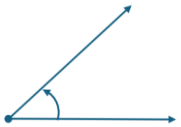


$$\text{if } \{\text{Point } L \text{ is between points } K \text{ and } M \text{ in } \overline{KM}\}$$
$$\text{then } \{\overline{KM} \cong \overline{KL} + \overline{LM} \rightarrow KM = KL + LM\}$$

Or

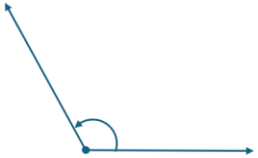
$$\text{if } \{\overline{KM} \cong \overline{KL} + \overline{LM} \rightarrow KM = KL + LM\}$$
$$\text{then } \{\text{Point } L \text{ is between points } K \text{ and } M \text{ in } \overline{KM}\}$$

Angle Classification



Acute Angle
$0^o < Angle < 90^o$

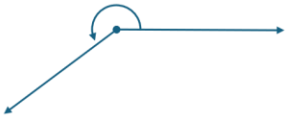
Right Angle $= 90^o$



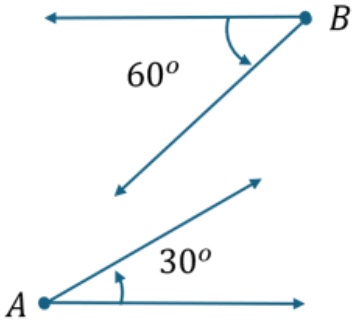
Obtuse Angle
$90^o < Angle < 180^o$

Straight or Plane Angle $= 180^o$

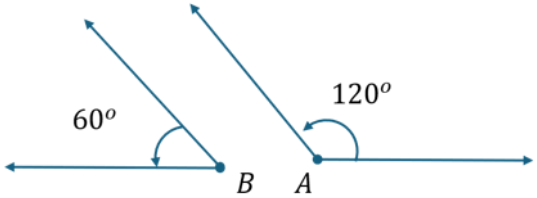
Reflex or Concave Angle
$180^o < Angle < 360^o$



Angle Pair Relationships

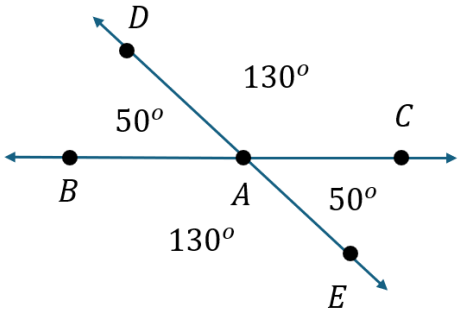


Complementary Angles
$m\angle A + m\angle B = 90^o$



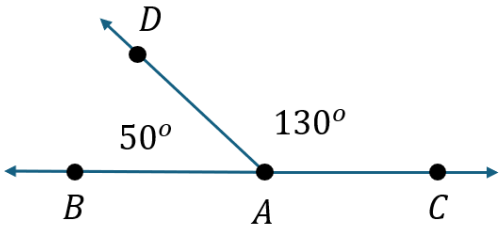
Supplementary Angles
$m\angle A + m\angle B = 180^o$

Angle Pair Relationships



$\angle BAD \cong \angle CAE$ & $\angle DAC \cong \angle EAB$

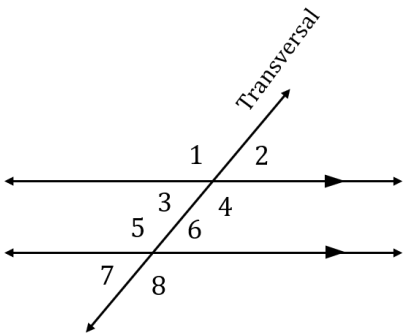
Vertical Angles:
OppoSite by the Vertex
Congruent Angles



$m\angle BAD + m\angle CAD = 180^o$

Linear Pair:
Supplementary Angles
Adjancent Angles
Common Ray $\overrightarrow{AD}$

Angle Pair Relationships in ∥Lines Cut by a Transversal



Corresponding Angles

$\angle 1 \cong \angle 5$
$\angle 2 \cong \angle 6$
$\angle 3 \cong \angle 7$
$\angle 4 \cong \angle 8$

Alternate Interior Angles

$\angle 3 \cong \angle 6$
$\angle 4 \cong \angle 5$

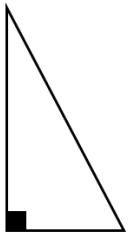
Alternate Exterior Angles

$\angle 1 \cong \angle 8$
$\angle 2 \cong \angle 7$

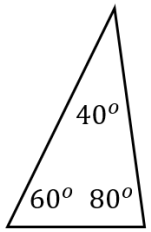
Same Side Interior Angles or Consecutive Interior Angles

$m\angle 3 + m\angle 5 = 180^o$ & $m\angle 4 + m\angle 6 = 180^o$

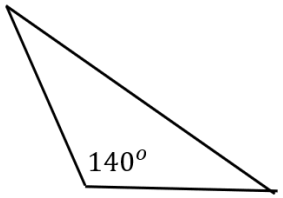
Triangle Clasification by Angles



Right Triangle: One Right Angle

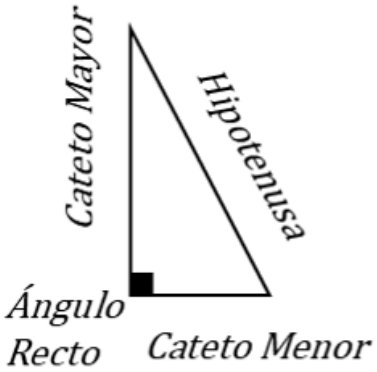


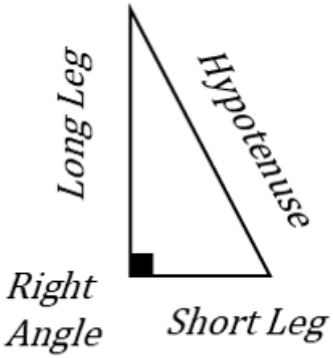
Acute Triangle: All Angles are Acute



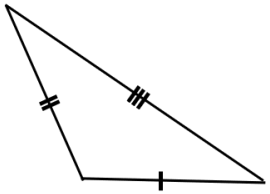
Obtuse Triangle: One Angle is Obtuse

Partes de un Triángulo Recto

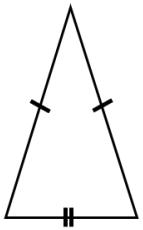




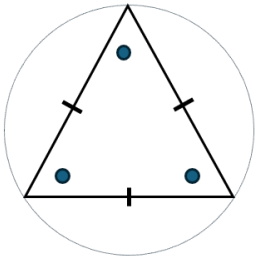
Triangle Clasification by Sides



Scalene Triangle: All Sides Different

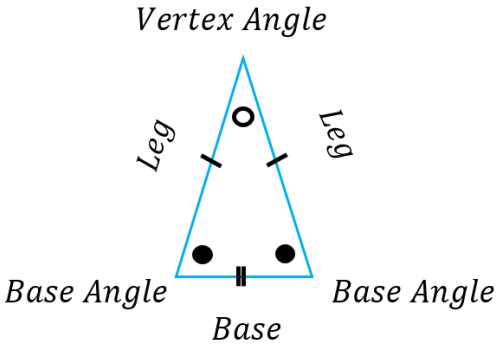


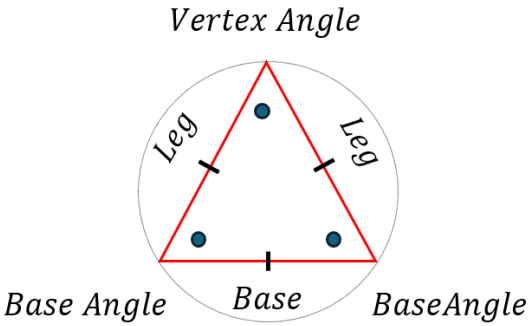
*Isosceles Triangle: **At Least** Two Congruent Sides*



Equilateral Triangle: Three Congruent Sides
● *Also Equiangular*

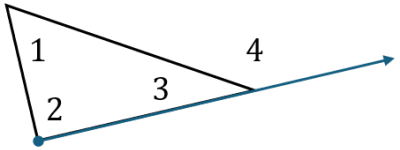
Parts of a Isosceles Triangle





All equilateral triangles are isosceles triangles but not all isosceles triangles are equilateral.

Exterior Angle Theorem



$\angle 1$ & $\angle 2$ → *Remote Inteior* $\angle$*'s*

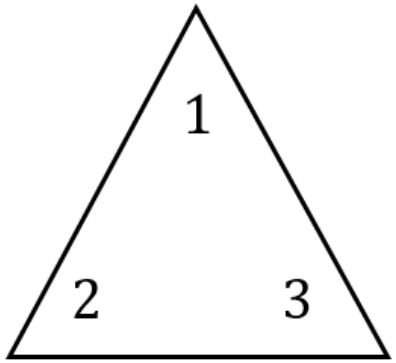
$\angle 3$ → *Interior No Adjacent* $\angle$ *to Exterior* $\angle$

$\angle 4$ → *Exterior* $\angle$

Exterior Angle is Equal to the Sum of Its Two Remote Interior Angles

$$m\angle 4 = m\angle 1 + m\angle 2$$

Interior Angle Sum in Triangles



$$m\angle 1 + m\angle 2 + m\angle 3 = 180^o$$

Proof of the Sum of Interior Angles in Regular and Irregular Concave Polygons

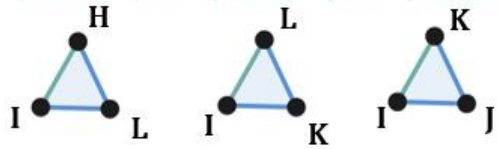
$$m\angle H + m\angle I + m\angle J + m\angle K + m\angle L = 540^o$$

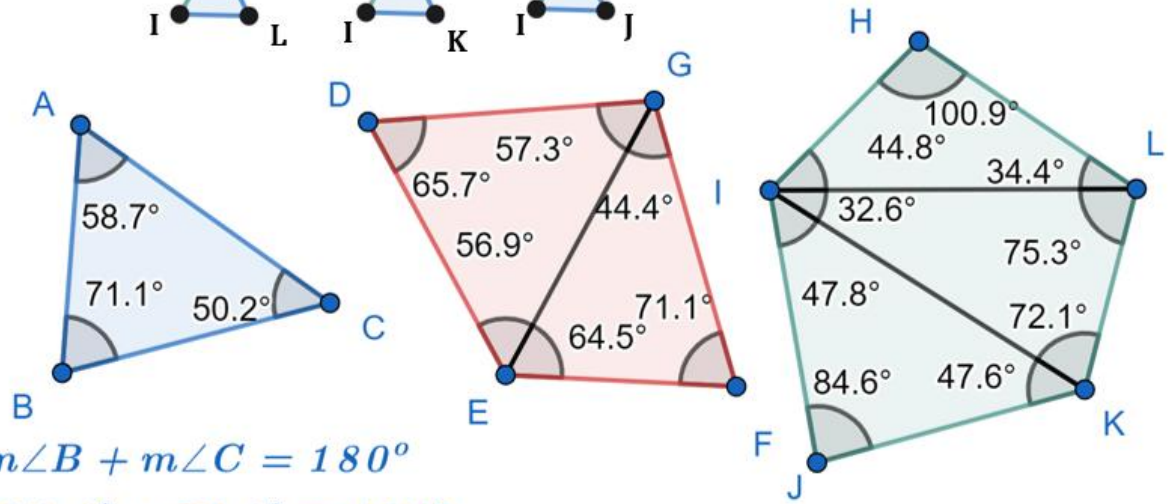
$$m\angle H + (m\angle LIH + m\angle KIL + m\angle JIK) + m\angle J + (m\angle LKI + m\angle IKJ) + (m\angle HLI + m\angle ILK) = 540^o$$

$$100.9^\circ + (44.8^\circ + 32.6^\circ + 47.8^\circ) + 84.6^\circ + (47.6^\circ + 72.1^\circ) + (75.3^\circ + 34.4^\circ) = 540^o$$

$$(100.9^\circ + 44.8^\circ + 34.4^\circ) + (32.6^\circ + 75.3^\circ + 72.1^\circ) + (47.6^\circ + 84.6^\circ + 47.8^\circ) = 540^o$$

$$(180^o) + (180^o) + (180^o) = 540^o$$





$$m\angle A + m\angle B + m\angle C = 180^o$$

$$58.7^\circ + 71.1^\circ + 50.2^\circ = 180^o$$

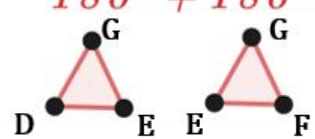
$$m\angle D + m\angle E + m\angle F + m\angle G = 360^o$$

$$m\angle D + (m\angle GED + m\angle FEG) + m\angle F + (m\angle DGE + m\angle EGF) = 360$$

$$65.7^\circ + (56.9^\circ + 57.3^\circ) + 71.1^\circ + (44.4^\circ + 64.5^\circ) = 360^o$$

$$(65.7^\circ + 56.9^\circ + 57.3^\circ) + (71.1^\circ\ 44.4^\circ + 64.5^\circ) = 360^o$$

$$180^o + 180^o = 360^o$$

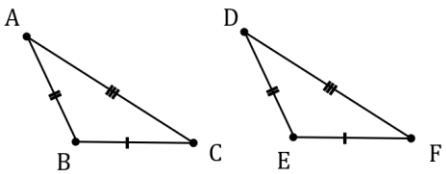


$S = \textit{Interior Angle Sum in Polygons}$
$= (\textit{Number of Triangles}) \cdot 180^o$

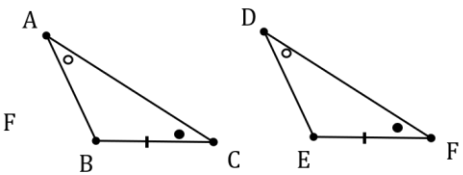
$\textit{Number Triangles} = \textit{Number Sides} - 2$
$n = \textit{Number Sides}$

$$\boxed{S = (n-2) \cdot 180^o}$$

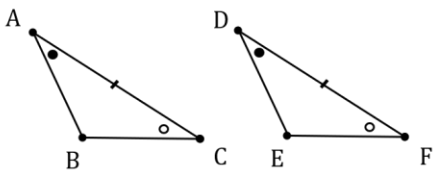
Triangle Congruence Theorems and Postulates



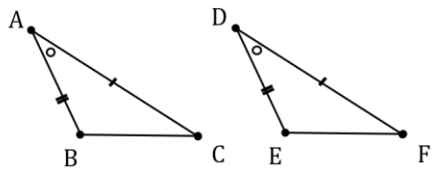
$\Delta ABC \cong \Delta DEF$
by Side – Side – Side or SSS



$\Delta ABC \cong \Delta DEF$
by Angle – Angle – Side or AAS

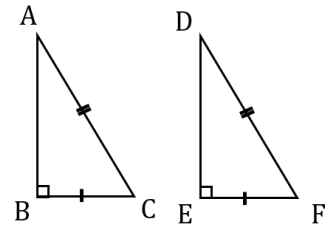


$\Delta ABC \cong \Delta DEF$
by Angle – Side – Angle or ASA

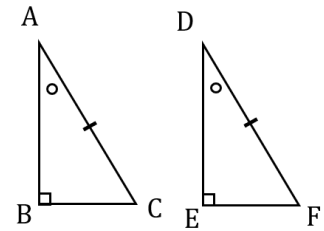


$\Delta ABC \cong \Delta DEF$
by Side – Angle – Side or SAS

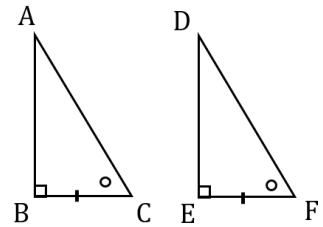
Triangle Clasification by Angles



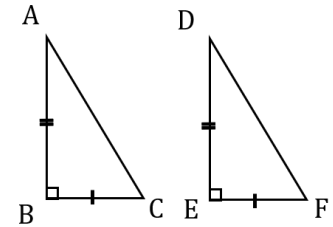
$\Delta ABC \cong \Delta DEF$
by Hypotenuse Leg ***or HL***



$\Delta ABC \cong \Delta DEF$
by Hypotenuse – Angle *or* ***HA***

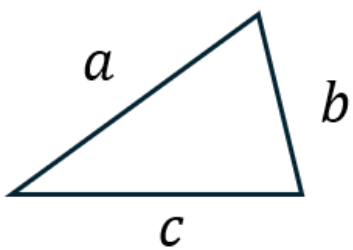


$\Delta ABC \cong \Delta DEF$
by Leg – Angle *or* ***LA***



$\Delta ABC \cong \Delta DEF$
by Leg – Leg *or* ***LL***

Triangle Inequality

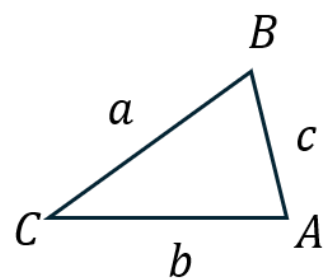


$$a < b + c$$

$$b < a + c$$

$$c < a + b$$

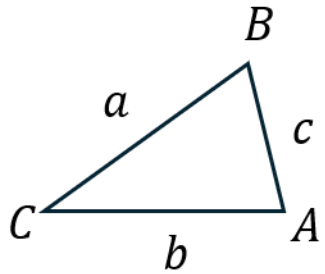
Triangle Inequality: Ordering Angles



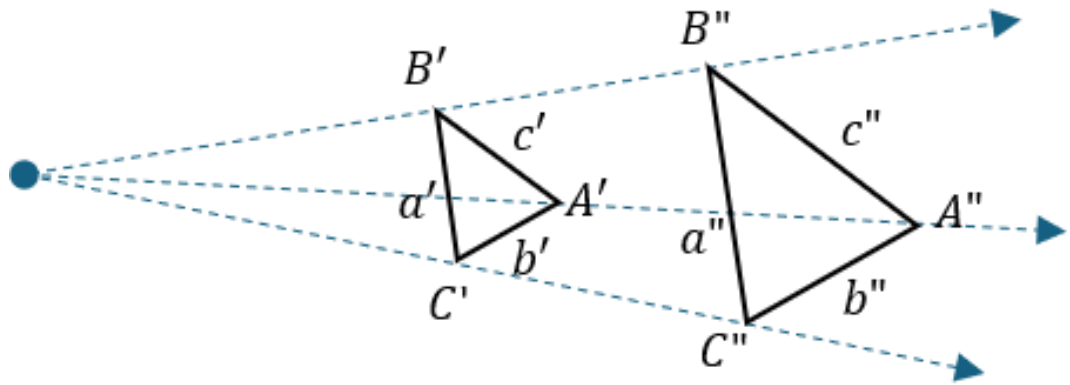
$$if\ \{a > b > c\}$$

$$then$$

$$\{m\angle A > m\angle B > m\angle C\}$$

Triangle Inequality*: *Ordering Sides	
 $if\ \{m\angle A > m\angle B > m\angle C\}$ $then$ $\{a > b > c\}$	

Similarity in Triangles



Corresponding Sides are Proportional
or
in Proportion

$$\boldsymbol{If}\left\{\begin{matrix}\dfrac{a'}{a''}=\dfrac{b'}{b''}=\dfrac{c'}{c''}\\ \boldsymbol{or}\\ \dfrac{a''}{a'}=\dfrac{b''}{b'}=\dfrac{c''}{c'}\end{matrix}\right\}\ \&\ \left\{\begin{matrix}\angle A'\cong\angle A''\\ \angle B'\cong\angle B''\\ \angle C'\cong\angle C''\end{matrix}\right\}$$

Corresponding Angles Congruent

$$\boldsymbol{Then}$$

$$\{\Delta A'B'C'\sim\Delta A''B''C''\}$$

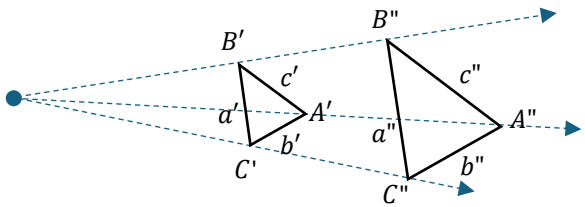
$\sim \quad \rightarrow$ ***Similarity Symbol***

The Converse is also true

Proportional Reasoning and Similarity in Triangles

Side−Side−Side or SSS ~

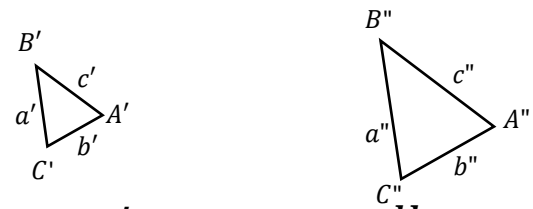
$$If \left\{ \begin{array}{c} \dfrac{a'}{a''} = \dfrac{b'}{b''} = \dfrac{c'}{c''} \\ or \\ \dfrac{a''}{a'} = \dfrac{b''}{b'} = \dfrac{c''}{c'} \end{array} \right\} \quad Then \left\{ \begin{array}{c} \Delta A'B'C' \sim \Delta A''B''C'' \\ by\ \ SSS\ \sim \end{array} \right\}$$



The converse is true as well

Angle − Angle ~

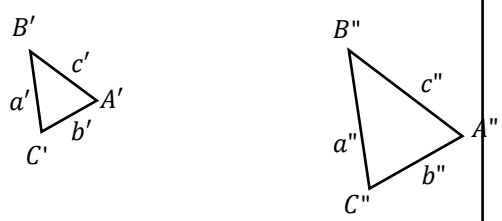
$$If \left\{ \begin{array}{c} \left\{ \begin{array}{c} \angle A' \cong \angle A'' \\ \angle B' \cong \angle B'' \end{array} \right\} \\ or \left\{ \begin{array}{c} \angle A' \cong \angle A'' \\ \angle C' \cong \angle C'' \end{array} \right\} \\ or \left\{ \begin{array}{c} \angle B' \cong \angle B'' \\ \angle C' \cong \angle C'' \end{array} \right\} \end{array} \right\} \quad Then \left\{ \begin{array}{c} \Delta A'B'C' \sim \Delta A''B''C'' \\ by\ \ AA\ \sim \end{array} \right\}$$



The converse is true as well

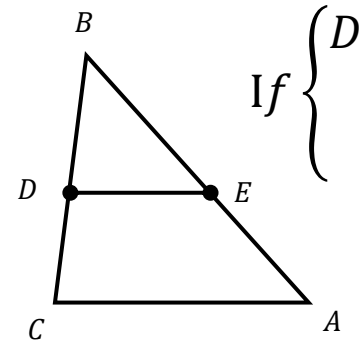
Side−Angle−Side or SAS ~

$$If \left\{ \begin{array}{c} \dfrac{a'}{a''} = \dfrac{c'}{c''} \\ or \\ \dfrac{a''}{a'} = \dfrac{c''}{c'} \\ and \\ \angle B' \cong \angle B'' \end{array} \right\} \quad Then \left\{ \begin{array}{c} \Delta A'B'C' \sim \Delta A''B''C'' \\ by\ \ SAS\ \sim \end{array} \right\}$$



The converse is true as well

Two Sides Connected at Midpoint in a Δ and the third one ∥ to this.

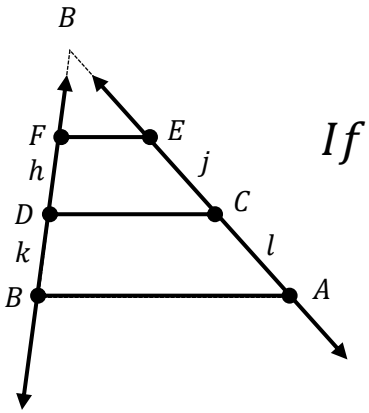


$$If \left\{ \begin{array}{c} D\ y\ E\ are\ midpoints \\ of \\ \overline{BC}\ and\ \overline{DA} \end{array} \right\} \quad Then \left\{ \begin{array}{c} \overline{DE}\ \parallel \overline{CA} \\ and \\ DE = \dfrac{1}{2} CA \end{array} \right\}$$

The converse is true as well

Proportional Reasoning and Similarity in Triangles

Porportionality in several ∥ lines cut by transversals

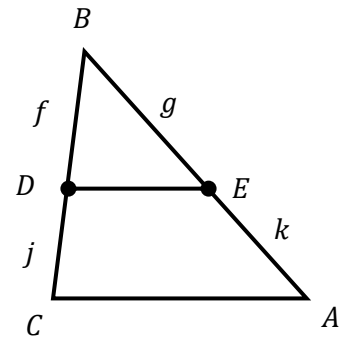


$$If \left\{ \begin{matrix} \overline{AB} \parallel \overline{CD} \\ and \\ \overline{CD} \parallel \overline{EF} \\ and \\ \overline{AB} \parallel \overline{EF} \end{matrix} \right\}$$

$$Then \left\{ \frac{k}{l} = \frac{h}{j} \right\}$$

The converse is true as well

Proportionality in two sides in a Δ joined by a segment ∥ to the third side.



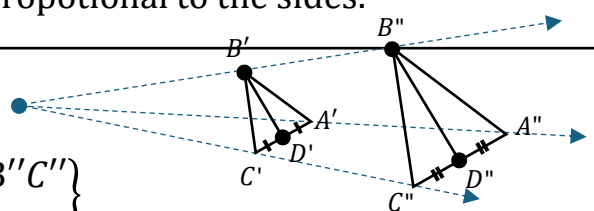
$$If \left\{ \overline{DE} \parallel \overline{CA} \right\}$$

$$Then$$

$$\left\{ \frac{f}{j} = \frac{g}{k} \right\}$$

The converse is true as well

Medians $\overline{B'D'}$ and $\overline{B''D''}$ in similar triangles are are propotional to the sides.



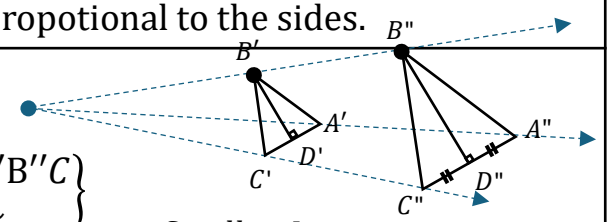
$$If \left\{ \begin{matrix} \Delta A'B'C' \sim \Delta A''B''C'' \\ are\ \Delta's \sim \end{matrix} \right\}$$

$$Smaller : Larger$$

$$Then \left\{ \begin{matrix} \dfrac{A'B'}{A''B''} = \dfrac{B'C'}{B''C''} = \dfrac{C'A'}{C''A''} = \dfrac{B'D'}{B''D''} \\ or \\ \dfrac{A''B''}{A'B'} = \dfrac{B''C''}{B'C} = \dfrac{C''A''}{C'A} = \dfrac{B''D''}{B'D'} \end{matrix} \right\}$$

$$Larger : Smaller$$

Altitudes or H*eights* $\overline{B'D'}$ and $\overline{B''D''}$ in similar triangles are are propotional to the sides.



$$If \left\{ \begin{matrix} \Delta A'B'C' \sim \Delta A''B''C \\ are\ \Delta's \sim \end{matrix} \right\}$$

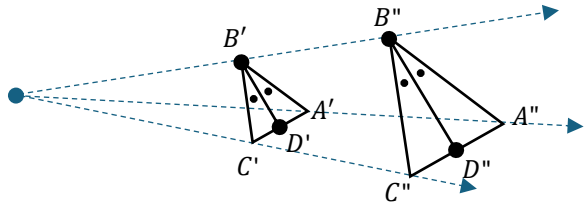
$$Smaller : Larger$$

$$Then \left\{ \begin{matrix} \dfrac{A'B'}{A''B''} = \dfrac{B'C'}{B''C''} = \dfrac{C'A'}{C''A''} = \dfrac{B'D'}{B''D''} \\ or \\ \dfrac{A''B''}{A'B'} = \dfrac{B''C''}{B'C} = \dfrac{C''A''}{C'A} = \dfrac{B''D''}{B'D'} \end{matrix} \right\}$$

$$Larger : Smaller$$

Proportional Reasoning and Similarity in Triangles

Angle Bisectors $\overline{B'D'}$ and $\overline{B"D"}$ in similar triangles are are propotional to the sides.



$$If \left\{ \begin{matrix} \Delta A'B'C' \sim \Delta A"B"C" \\ are\ \Delta's \sim \end{matrix} \right\}$$

$$Smaller : Larger$$

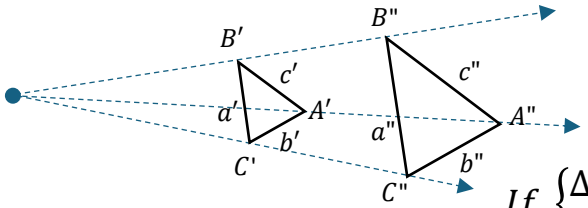
$$Then \left\{ \begin{matrix} \dfrac{A'B'}{A"B"} = \dfrac{B'C'}{B"C"} = \dfrac{C'A'}{C"A"} = \dfrac{B'D'}{B"D"} \\ or \\ \dfrac{A"B"}{A'B'} = \dfrac{B"C"}{B'C} = \dfrac{C"A"}{C'A} = \dfrac{B"D"}{B'D'} \end{matrix} \right\}$$

$$Larger : Smaller$$

Perimeters

$a' + b' + c'$ and $a" + b" + c"$

in similar triangles are are propotional to the sides.



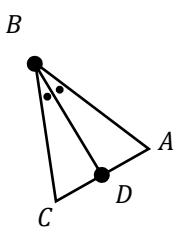
$$If \left\{ \begin{matrix} \Delta A'B'C' \sim \Delta A"B"C" \\ are\ \Delta's \sim \end{matrix} \right\}$$

$$Smaller : Larger$$

$$Then \left\{ \begin{matrix} \dfrac{A'B'}{A"B"} = \dfrac{B'C'}{B"C"} = \dfrac{C'A'}{C"A"} = \dfrac{a' + b' + c'}{a" + b" + c"} \\ or \\ \dfrac{A"B"}{A'B'} = \dfrac{B"C"}{B'C} = \dfrac{C"A"}{C'A} = \dfrac{a" + b" + c"}{a' + b' + c'} \end{matrix} \right\}$$

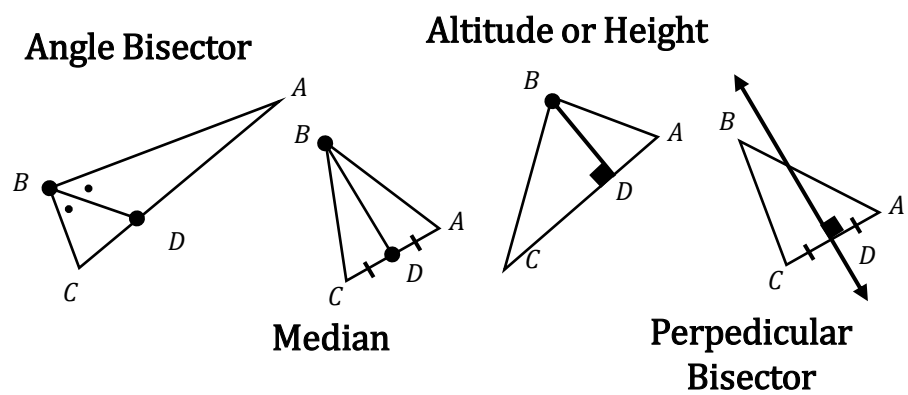
$$Larger : Smaller$$

Segments Formed by Angle Bisector from One Vertex in Opposite Side are Proportional to Corresponding Adjacent Sides

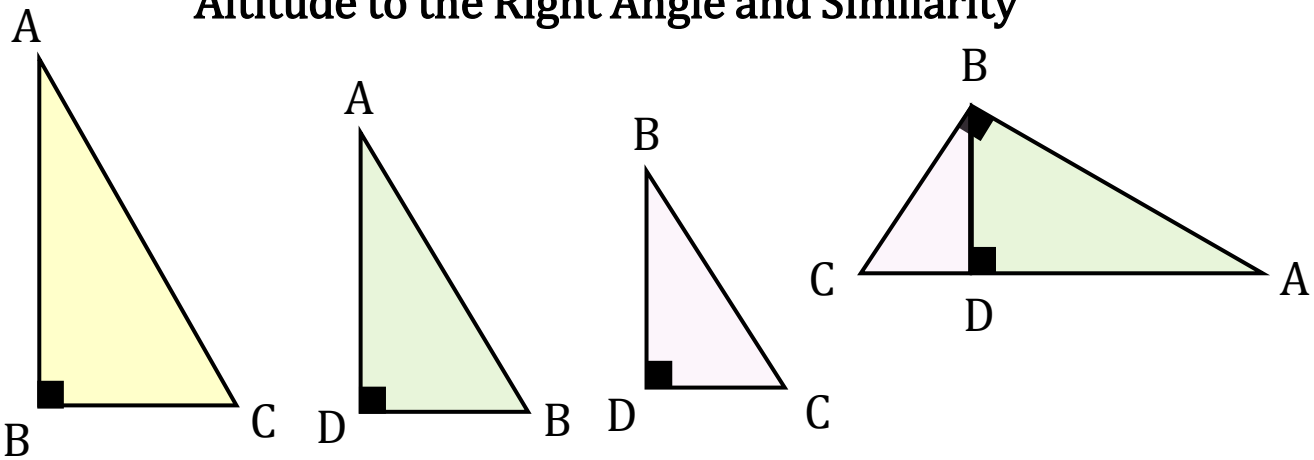


$$\left\{ \frac{CD}{AD} = \frac{BC}{BA} \right\}$$

Special Segments, Lines, Rays in Triangles



Altitude to the Right Angle and Similarity



The highlighted proportions involve geometric means; knowing that the product of the means equals the product of the extremes, we derive the three geometric mean theorems associated with the altitude to the right angle.

$\Delta ABC \sim \Delta ADB$

$$\frac{\boldsymbol{AB}}{\boldsymbol{AD}} = \frac{\boldsymbol{AC}}{\boldsymbol{AB}}$$

$$\frac{AB}{AD} = \frac{BC}{DB}$$

$$\frac{AC}{AB} = \frac{BC}{DB}$$

$$\frac{\boldsymbol{AB}}{\boldsymbol{AD}} = \frac{\boldsymbol{AC}}{\boldsymbol{AB}} \rightarrow \boldsymbol{AB^2 = AC \cdot AD}$$

The square of the geometric mean of the longer leg is equal to the product of the hypotenuse and the adjacent segment of the hypotenuse formed by the altitude to the right angle.

$\Delta ADB \sim \Delta BDC$

$$\frac{AD}{DB} = \frac{AB}{BC}$$

$$\frac{\boldsymbol{AD}}{\boldsymbol{DB}} = \frac{\boldsymbol{DB}}{\boldsymbol{DC}}$$

$$\frac{AB}{BC} = \frac{DB}{DC}$$

$$\frac{\boldsymbol{AD}}{\boldsymbol{DB}} = \frac{\boldsymbol{DB}}{\boldsymbol{DC}} \rightarrow \boldsymbol{DB^2 = AD \cdot DC}$$

The square of the geometric mean of the altitude to the right angle is equal to the product of the segments of the hypotenuse formed by the altitude to the right angle.

$\Delta ABC \sim \Delta BDC$

$$\frac{AB}{BD} = \frac{AC}{BC}$$

$$\frac{AB}{BD} = \frac{BC}{DC}$$

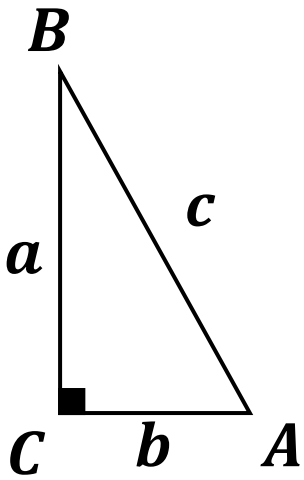
$$\frac{\boldsymbol{AC}}{\boldsymbol{BC}} = \frac{\boldsymbol{BC}}{\boldsymbol{DC}}$$

$$\frac{\boldsymbol{AC}}{\boldsymbol{BC}} = \frac{\boldsymbol{BC}}{\boldsymbol{DC}} \rightarrow \boldsymbol{BC^2 = AC \cdot DC}$$

The square of the geometric mean of the shorter leg is equal to the product of the hypotenuse and the adjacent segment of the hypotenuse formed by the altitude to the right angle.

Right Triangles: Pythagorean Theorem

Pythagorean theorem



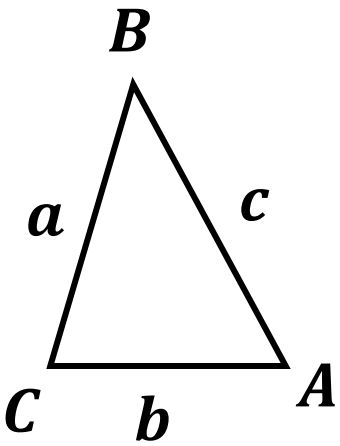
$$c^2 = a^2 + b^2$$

$$|c| = \sqrt{a^2 + b^2}$$

$$c = \pm\sqrt{a^2 + b^2}$$

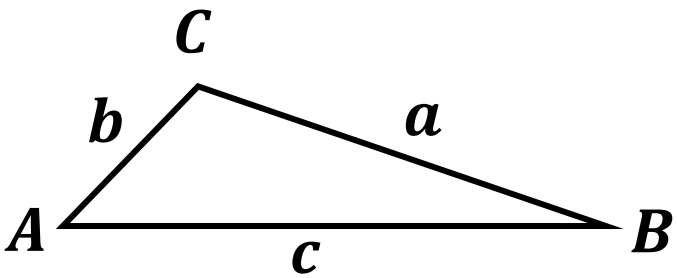
$(-)$ *No sense here*

Pythagorean theorem *Acute* $\Delta's$



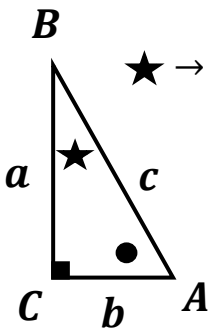
$$c^2 < a^2 + b^2$$

Pythagorean theorem *Obtuse* $\Delta's$

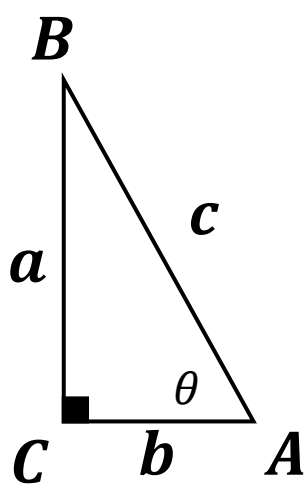


$$c^2 > a^2 + b^2$$

Right Triangles: Trigonometry

Trigonometry Reference Angle	Trigonometry: Right Triangles Bold text used at higher levels
● → ***Reference Angle*** ***a*** → ***Opposite Side*** ***b*** → ***Adjacent Side*** ★ → ***Reference Angle*** ***b*** → ***Opposite Side*** ***a*** → ***Adjacent Side*** 	● → ***Reference Angle*** ★ → ***Reference Angle*** $\sin A = \frac{a}{c}$ $\quad \mathbf{\cot A = \frac{b}{a}}$ $\quad \sin B = \frac{b}{c}$ $\quad \mathbf{\cot B = \frac{a}{b}}$ $\cos A = \frac{b}{c}$ $\quad \mathbf{\sec A = \frac{c}{b}}$ $\quad \cos B = \frac{a}{c}$ $\quad \mathbf{\sec B = \frac{c}{a}}$ $\tan A = \frac{a}{b}$ $\quad \mathbf{\csc A = \frac{c}{a}}$ $\quad \tan B = \frac{b}{a}$ $\quad \mathbf{\csc B = \frac{c}{b}}$

Trigonometría: Triángulos Rectos



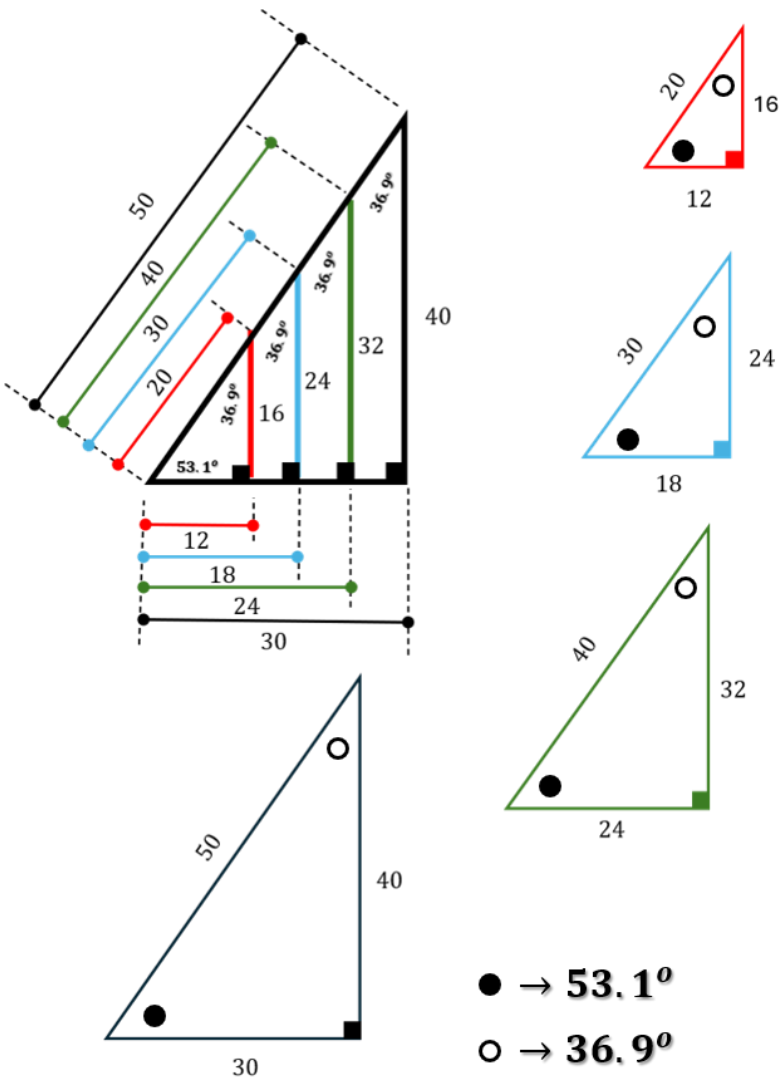
$\alpha, \beta, \gamma, \delta, \theta \rightarrow$ *It is common to use them as angles.*

$$\sin\theta = \frac{Opposite\ Side}{Hypotenuse} \qquad \cot\theta = \frac{Adjacent\ Side}{Opposite\ Side}$$

$$\cos\theta = \frac{Adjacent\ Side}{Hypotenuse} \qquad \sec\theta = \frac{Hypotenuse}{Adjacent\ Side}$$

$$\tan\theta = \frac{Opposite\ Side}{Adjacent\ Side} \qquad \csc\theta = \frac{Hypotenuse}{Opposite\ Side}$$

Establishing the similarity of triangles that have congruent corresponding angles and corresponding sides that are proportional, or relating this to trigonometric ratios or trigonometry.



● → 53.1^o

○ → 36.9^o

Similarity Proof

$$\sin(53.1^o) \approx \frac{16}{20} = \frac{24}{30} = \frac{32}{40} = \frac{40}{50} = \frac{4}{5} \approx 0.7997$$

$$\cos(53.1^o) \approx \frac{12}{20} = \frac{18}{30} = \frac{24}{40} = \frac{30}{50} = \frac{3}{5} \approx .6004$$

$$\tan(53.1^o) \approx \frac{16}{12} = \frac{24}{18} = \frac{32}{24} = \frac{40}{30} = \frac{4}{3} \approx 1.332$$

$$\cot(53.1^o) \approx \frac{12}{16} = \frac{18}{24} = \frac{24}{32} = \frac{30}{40} = \frac{3}{4} \approx 0.7508$$

$$\sec(53.1^o) \approx \frac{20}{12} = \frac{30}{18} = \frac{40}{24} = \frac{50}{30} = \frac{5}{3} \approx 1.666$$

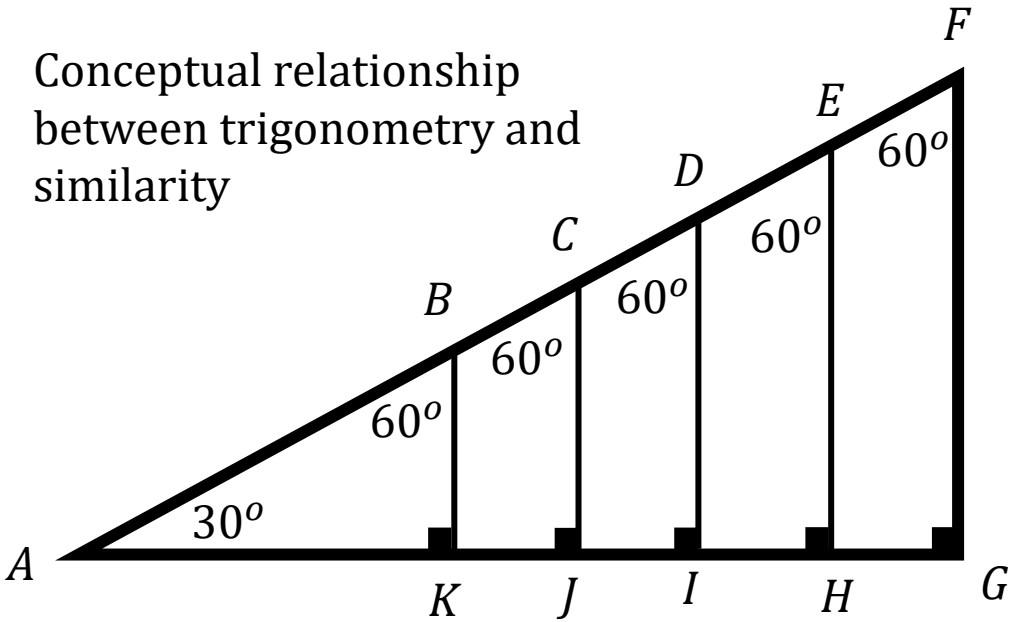
$$\csc(53.1^o) = \frac{20}{16} = \frac{30}{24} = \frac{40}{32} = \frac{50}{40} = \frac{5}{4} = 1.25$$

$$\boxed{\text{All } \Delta's \text{ are } (\sim) \text{ by AA} \sim}$$

Right Triangles: Trigonometry and Similatiry Link

Trigonom*etría de Triángulos Rectos y* (~)

Conceptual relationship between trigonometry and similarity



$\boldsymbol{\Delta ABK \sim \Delta ACJ \sim \Delta ADI \sim \Delta AEH \sim \Delta AFG}$

All embedded triangles are similar.

$\mathbf{30^o - 60^0 - 90^o}$

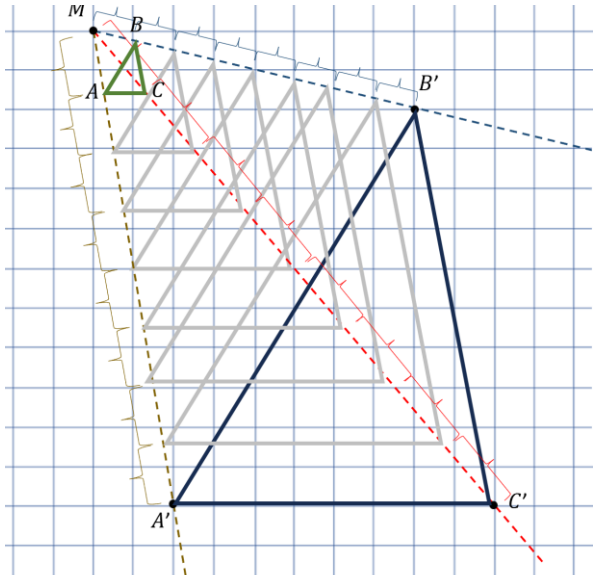
Scalene right triangles.

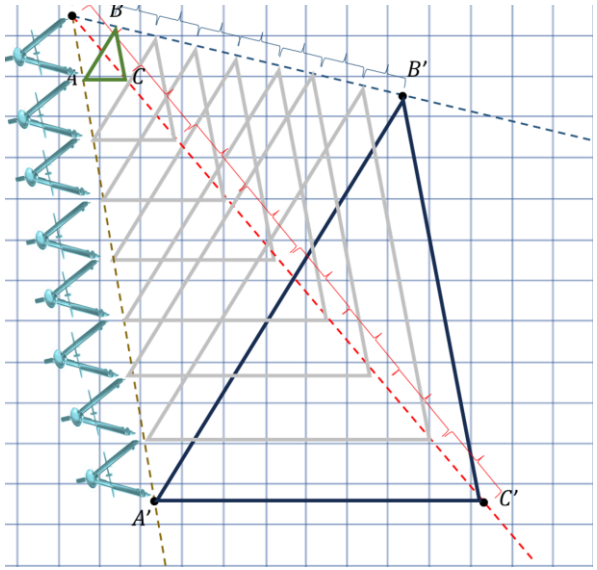
They all yield the same numerical value for each of the six trigonometric ratios (**angle-to-ratio**) and all produce the same corresponding angle (**ratio-to-angle**).

Angle to Trigonometric Ratio	**Trigonometric Ratio to Angle**
$\sin(\mathbf{30^o}) = 0.5$	$\sin^{-1}(\mathbf{0.5}) = 30^o$
$\cos(\mathbf{30^o}) \approx 0.87$	$\cos^{-1}(\mathbf{0.87}) \approx 30^o$
$\tan(\mathbf{30^o}) \approx 0.58$	$tan^{-1}(\mathbf{0.58}) \approx 30^o$

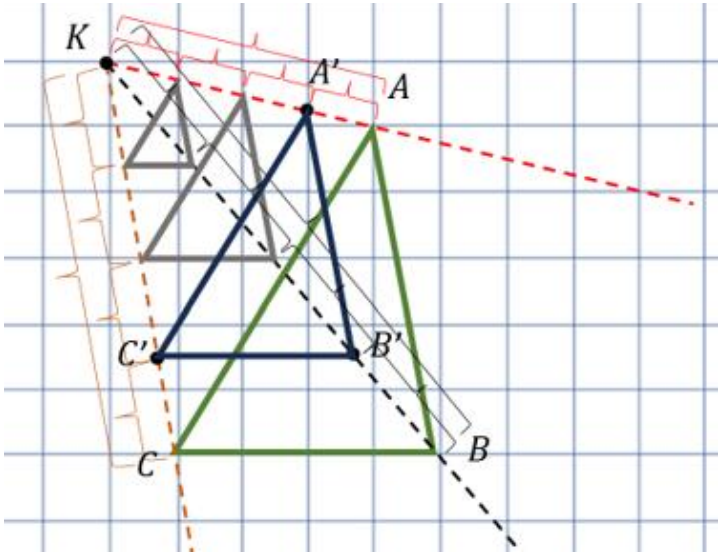
Similarity vs Homothety: Enlargements and Reductions

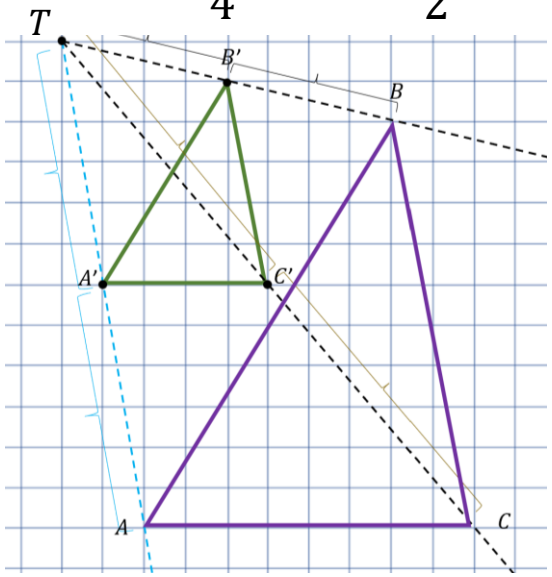
Perfoming an elongation by a factor of 8 with center of dilation at point M





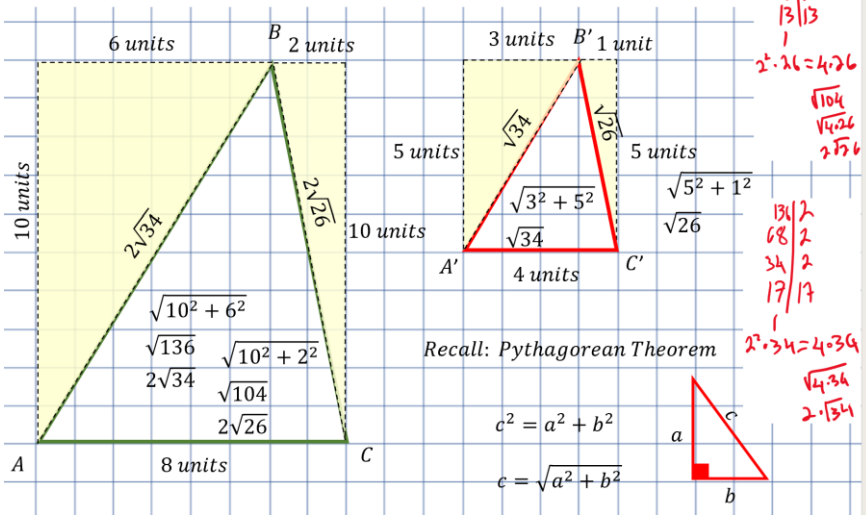
Perfoming Fractrional Dilations: Reduction to $\frac{3}{4}$ *and to* $\frac{1}{2}$





Similarity: Establishing Extended Proportions and Corresponding Angles $\cong$' s

Last problem we proved how to get a fractional dilation with a scale of $1:\frac{1}{2}$ with center of dilation T. Now let's prove they are similar with corresponding sides proportional and corresponding angles congruent (Angles are with the same orientation)



104 | 2
52 | 2
26 | 2
13 | 13
1
$2^2 \cdot 26 = 4 \cdot 26$
$\sqrt{104}$
$\sqrt{4 \cdot 26}$
$2\sqrt{26}$

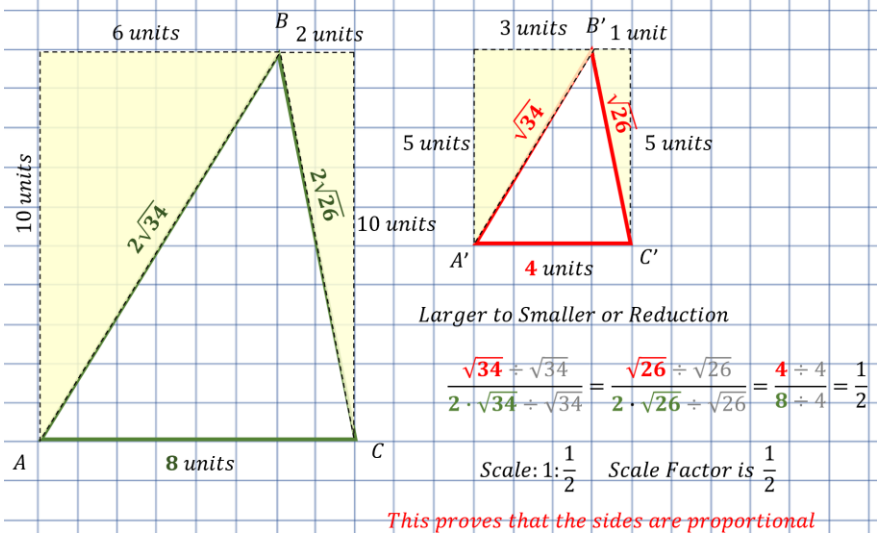
136 | 2
68 | 2
34 | 2
17 | 17
1
$2^2 \cdot 34 = 4 \cdot 34$
$\sqrt{4 \cdot 34}$
$2 \cdot \sqrt{34}$

Recall: Pythagorean Theorem

$$c^2 = a^2 + b^2$$

$$c = \sqrt{a^2 + b^2}$$

Last problem we proved how to get a fractional dilation with a scale of $1:\frac{1}{2}$ with center of dilation T. Now let's prove they are similar with corresponding sides proportional and corresponding angles congruent (Angles are with the same orientation)



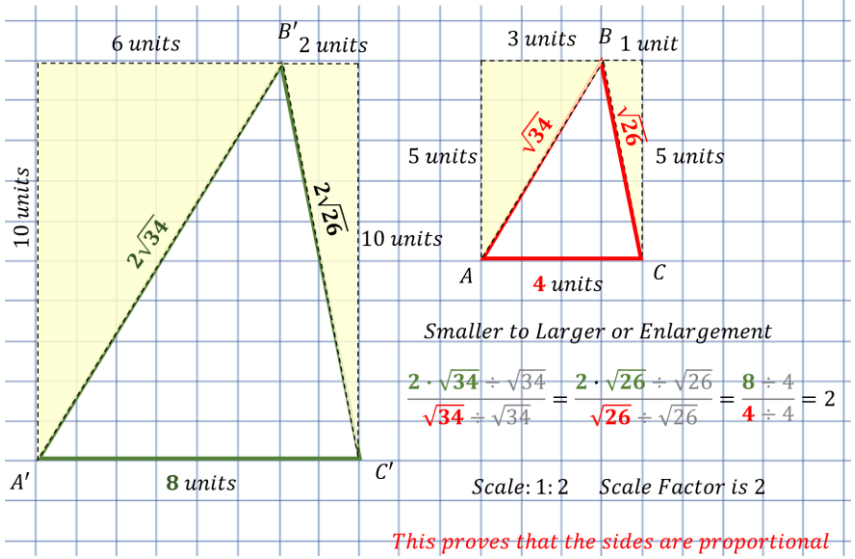
Larger to Smaller or Reduction

$$\frac{\sqrt{34} \div \sqrt{34}}{2 \cdot \sqrt{34} \div \sqrt{34}} = \frac{\sqrt{26} \div \sqrt{26}}{2 \cdot \sqrt{26} \div \sqrt{26}} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2}$$

Scale: $1:\frac{1}{2}$ Scale Factor is $\frac{1}{2}$

This proves that the sides are proportional

Last problem we proved how to get a fractional dilation with a scale of $1:\frac{1}{2}$ with center of dilation T. Now let's prove they are similar with corresponding sides proportional and corresponding angles congruent (Angles are with the same orientation)



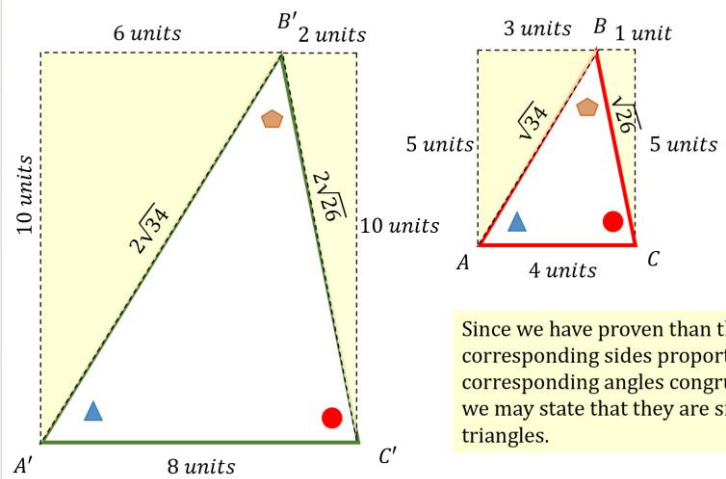
Smaller to Larger or Enlargement

$$\frac{2 \cdot \sqrt{34} \div \sqrt{34}}{\sqrt{34} \div \sqrt{34}} = \frac{2 \cdot \sqrt{26} \div \sqrt{26}}{\sqrt{26} \div \sqrt{26}} = \frac{8 \div 4}{4 \div 4} = 2$$

Scale: 1: 2 Scale Factor is 2

This proves that the sides are proportional

Last problem we proved how to get a fractional dilation with a scale of $1:\frac{1}{2}$ with center of dilation T. Now let's prove they are similar with corresponding sides proportional and corresponding angles congruent (Angles are with the same orientation)



Since we have proven than they have corresponding sides proportional and corresponding angles congruent then we may state that they are similar triangles.

At the last slides of this lesson, you will find how to prove corresponding angles congruent using Law of Cosines and Law of Sines as an enrichment extension

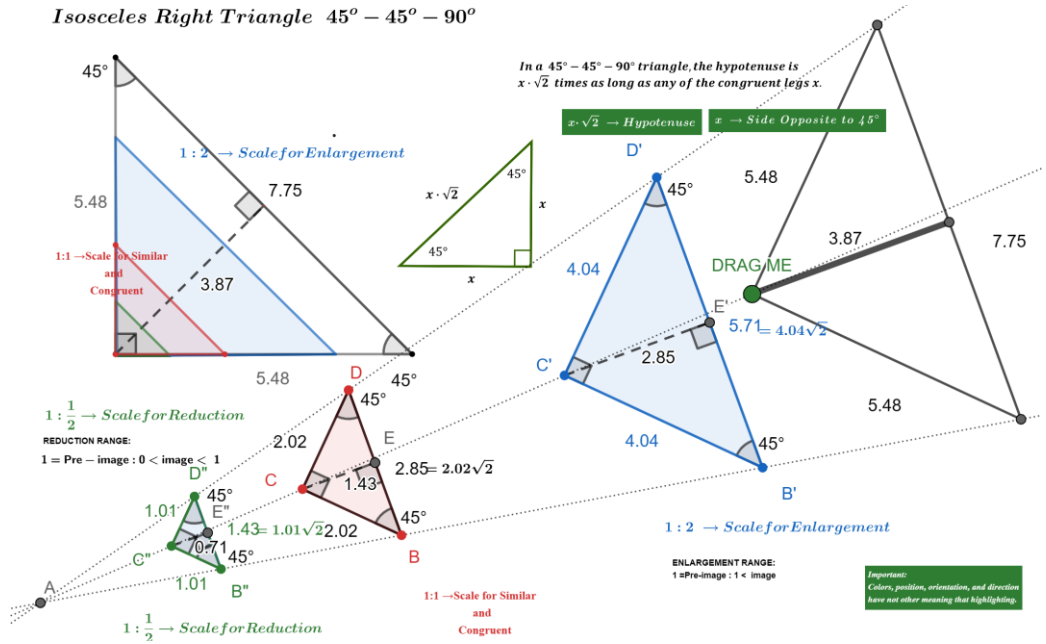
$$\Delta ABC \sim \Delta A'B'C'$$

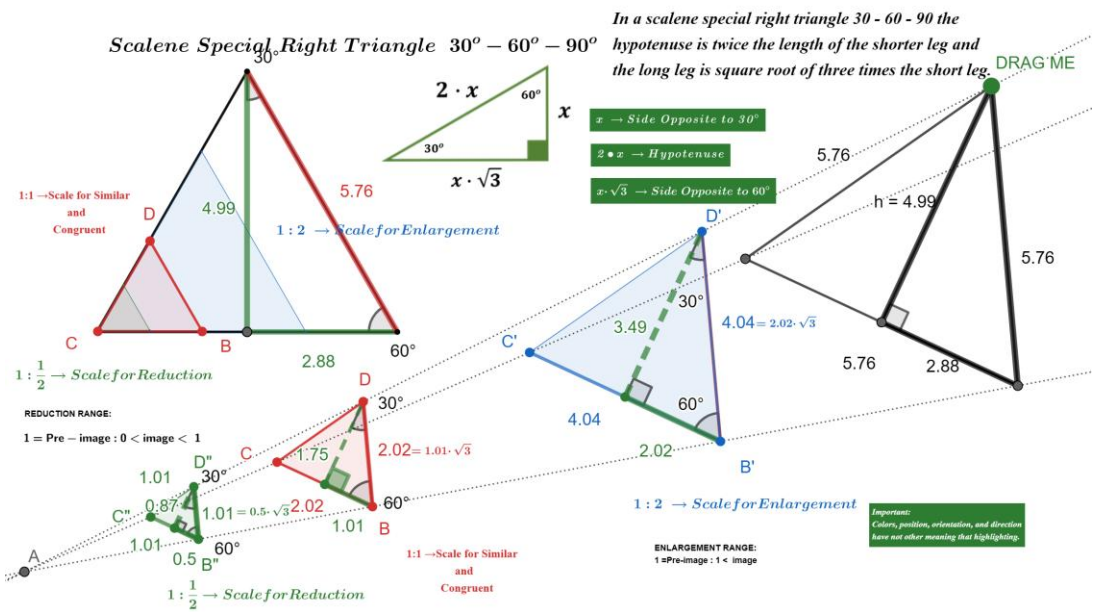
Similarity: Homothety from a center of homothecy or dilation for special triangles

Several instances of a special triangle

$$45^o - 45^o - 90^o$$

using a center of homothety, distinguishing between enlargement and reduction, and presenting cases of congruence and similarity



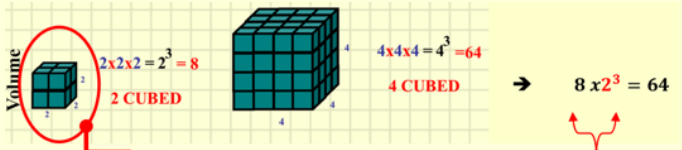


Several instances of a special triangle

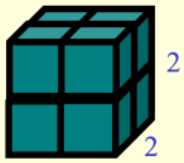
$$30^o - 60^o - 90^o$$

using a center of homothety, distinguishing between enlargement and reduction, and presenting cases of congruence and similarity

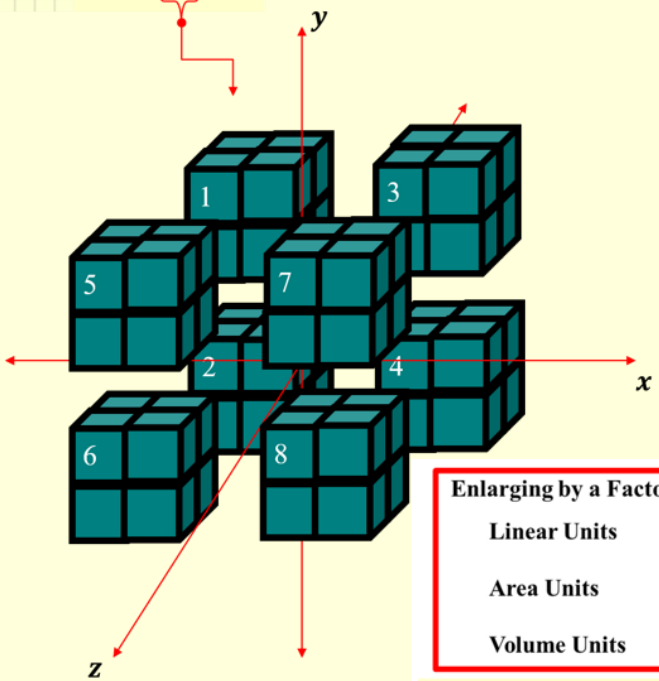
Scale factor in Similarity or using it in non-congruent transformations



➔ $8\,x2^3 = 64$



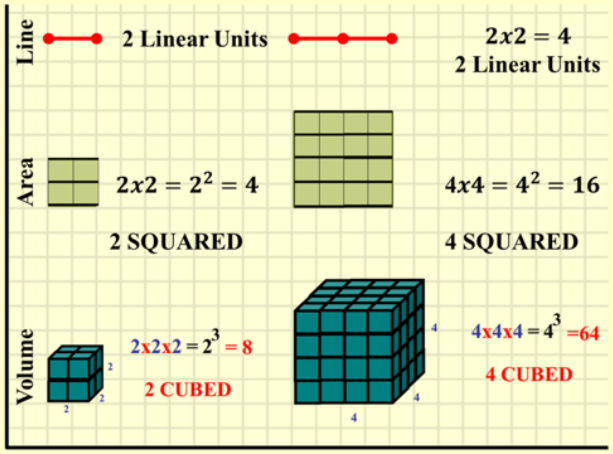
$2^3 = 2 \cdot 2 \cdot 2 = 8$



It is necessary to take into account that making dilations of line dimensions is different of doing it of area units or volumen units.
Special considerations need to be done if we are going to use as scale factor linear, area or volume units

Enlarging by a Factor of "a"		
Linear Units	➔	Multiply by a
Area Units	➔	Multiply by a^2
Volume Units	➔	Multiply by a^3

Enlargements or reductions of a line, an area, or a volume need to be done as indicated in the left. You multiply by the factor of enlargement to the first power for linear units, to the second power for areas, and to the third power for volumes.



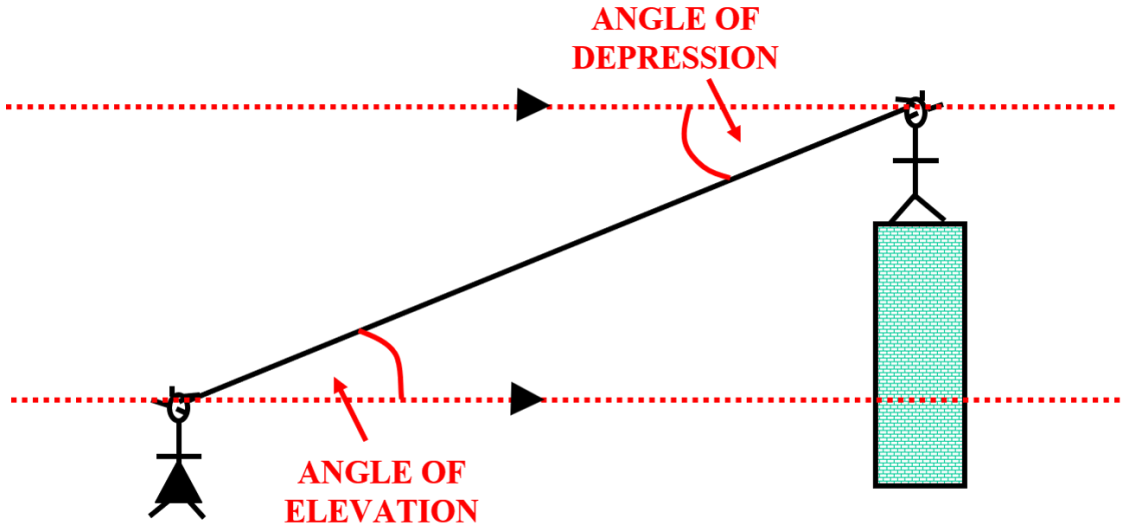
➔ $2\,x2 = 4$

➔ $4\,x2^2 = 16$

➔ $8\,x2^3 = 64$

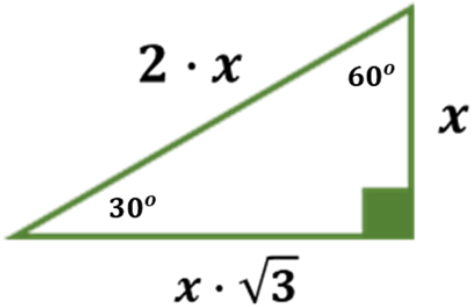
Enlargements by a Factor of "2"

Angles of Depression and Elevation used in Word problems for Trigonometry



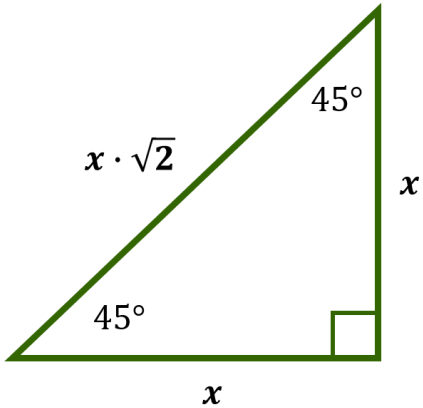
Angles of Depression and Elevation are Alternate Interior Angles and they are congruent.

Scalene *Special Right Triangle*: $30° - 60° - 90°$



In a $30° - 60° - 90°$ *triangle, the hypotenuse is twice as long as the shorter leg* $2 \cdot x$ *and the longer leg is* $x \cdot \sqrt{3}$ *times as long as the shorter leg.*

Isosceles Special Right Triangle: $45° - 45° - 90°$

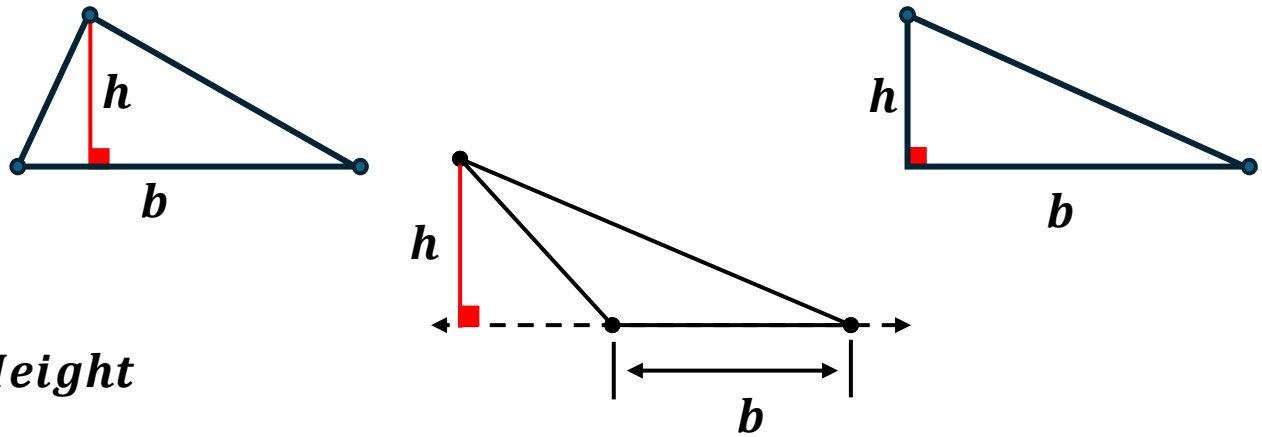


In a $45° - 45° - 90°$ *triangle, the hypotenuse is* $x \cdot \sqrt{2}$ *times as long as any of the congruent legs* x.

Polygons: Areas

Area of a Triangle

$$A = \frac{1}{2} \cdot b \cdot h$$



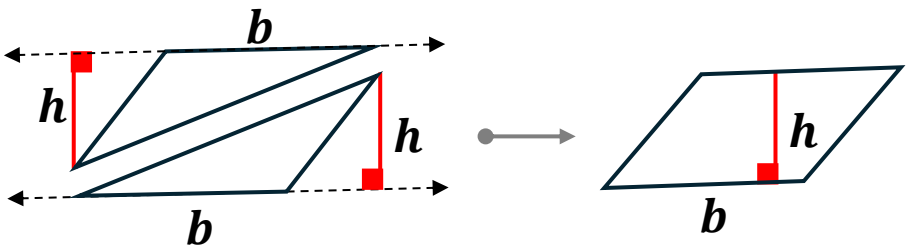
Where:

$A = Area$

$b = Base$

$h = Altitude\ or\ Height$

Area of a Parallelogram



$$A = \frac{1}{2} \cdot b \cdot h + \frac{1}{2} \cdot b \cdot h$$

$$A = b \cdot h\left(\frac{1}{2} + \frac{1}{2}\right) = b \cdot h$$

$$A = \boldsymbol{b \cdot h}$$

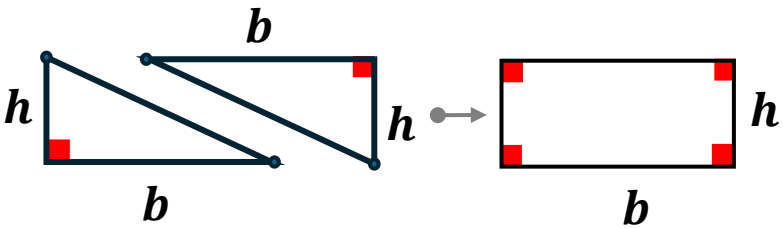
Where:

$A = Area$

$b = Base$

$h = Altitude\ or\ Height$

Area of a Rectangle



$$A = \frac{1}{2} \cdot b \cdot h + \frac{1}{2} \cdot b \cdot h$$

$$A = b \cdot h\left(\frac{1}{2} + \frac{1}{2}\right) = b \cdot h$$

$$A = \boldsymbol{b \cdot h}$$

Where:

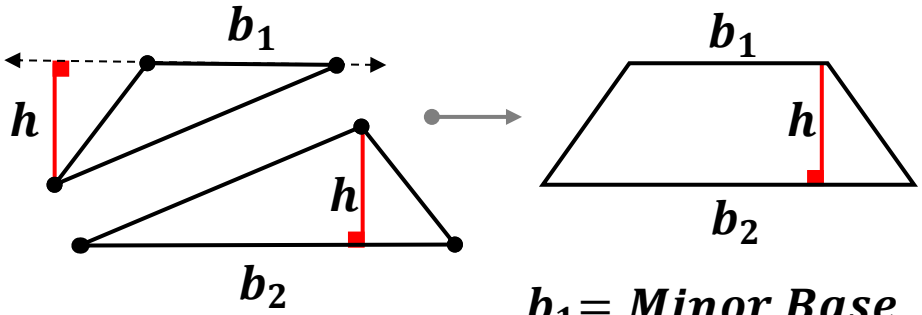
$A = Area$

$b = Base$

$h = Altitude\ or\ Height$

Polygons: Areas

Area of a Trapezoid



$\boldsymbol{A = Area}$

$\boldsymbol{h = height}$

$$A = \frac{1}{2} \cdot \boldsymbol{b_1} \cdot h + \frac{1}{2} \cdot \boldsymbol{b_2} \cdot h$$

$$\boldsymbol{A = \frac{1}{2} \cdot h(b_1 + b_2)}$$

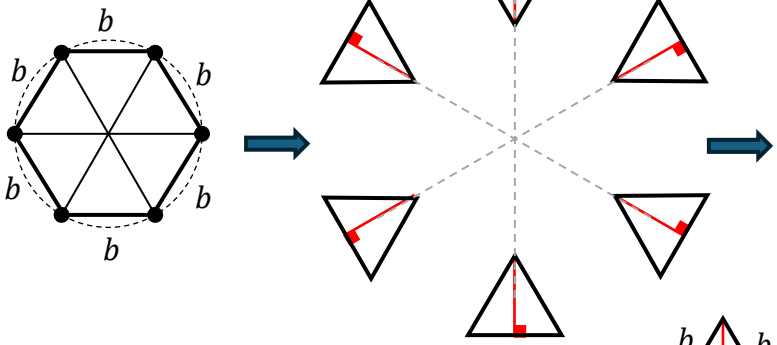
$\boldsymbol{b_1 = Minor\ Base}$

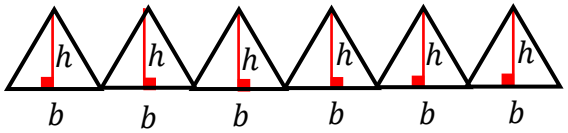
$\boldsymbol{b_2 = Major\ Base}$

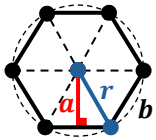
$\boldsymbol{M = Median}$

$$\boldsymbol{M = \frac{1}{2} \cdot (b_1 + b_2)}$$

Area of a Regular Convex Polygon (True for all)

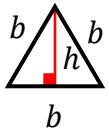






$$A = \frac{1}{2} \cdot hb + \frac{1}{2} \cdot hb + \frac{1}{2} \cdot hb + \frac{1}{2} \cdot hb + \frac{1}{2} \cdot hb + \frac{1}{2} \cdot hb$$

$$\rightarrow \quad A = \frac{1}{2} \cdot h(b + b + b + b + b + b) = \frac{1}{2} \cdot P \cdot h$$



$\rightarrow$ *Equilateral* Δ *with Side "b"*

If {Regular Convex Polygon}

then $\{\boldsymbol{h = height \rightarrow a = Apothem}\}$

$$Area = \boldsymbol{\frac{1}{2} \cdot P \cdot a}$$

$\boldsymbol{Perimeter_{Regular\ Convex\ Polygon}}$

$$P = b + b + b + b + b + b$$

Rhombus Area:

$$\boldsymbol{A = \frac{1}{2} \cdot (d_1 \cdot d_2)}$$

Where:

$A = Area$

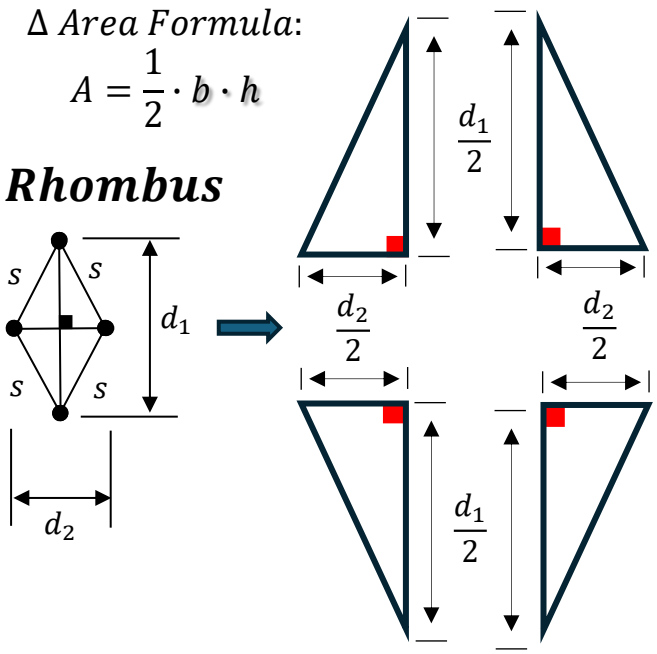
$d_1 = Large\ Diagonal$

$d_2 = Small\ Diagonal$

$\Delta\ Area\ Formula$:

$$A = \frac{1}{2} \cdot b \cdot h$$

Rhombus



$$A = \frac{1}{2} \cdot \left(\frac{d_1}{2} \cdot \frac{d_2}{2}\right) + \frac{1}{2} \cdot \left(\frac{d_1}{2} \cdot \frac{d_2}{2}\right) + \frac{1}{2} \cdot \left(\frac{d_1}{2} \cdot \frac{d_2}{2}\right) + \frac{1}{2} \cdot \left(\frac{d_1}{2} \cdot \frac{d_2}{2}\right)$$

$$A = 4 \cdot \left[\frac{1}{2} \cdot \left(\frac{d_1}{2} \cdot \frac{d_2}{2}\right)\right]$$

$$= \frac{4}{2} \cdot \left(\frac{d_1}{2} \cdot \frac{d_2}{2}\right)$$

$$= 2 \cdot \left(\frac{d_1}{2} \cdot \frac{d_2}{2}\right) = \boldsymbol{\frac{1}{2} \cdot (d_1 \cdot d_2)}$$

Area Circle:

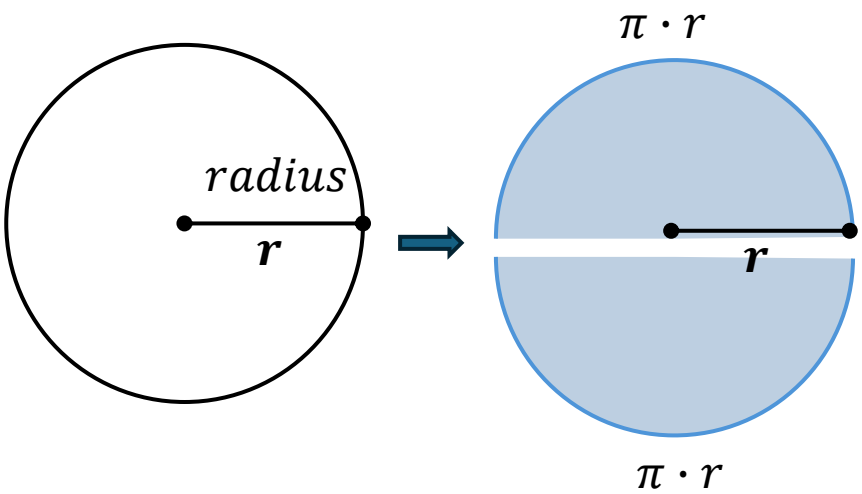
$$\boldsymbol{A = \pi \cdot r^2}$$

Perimeter of a $\odot$:

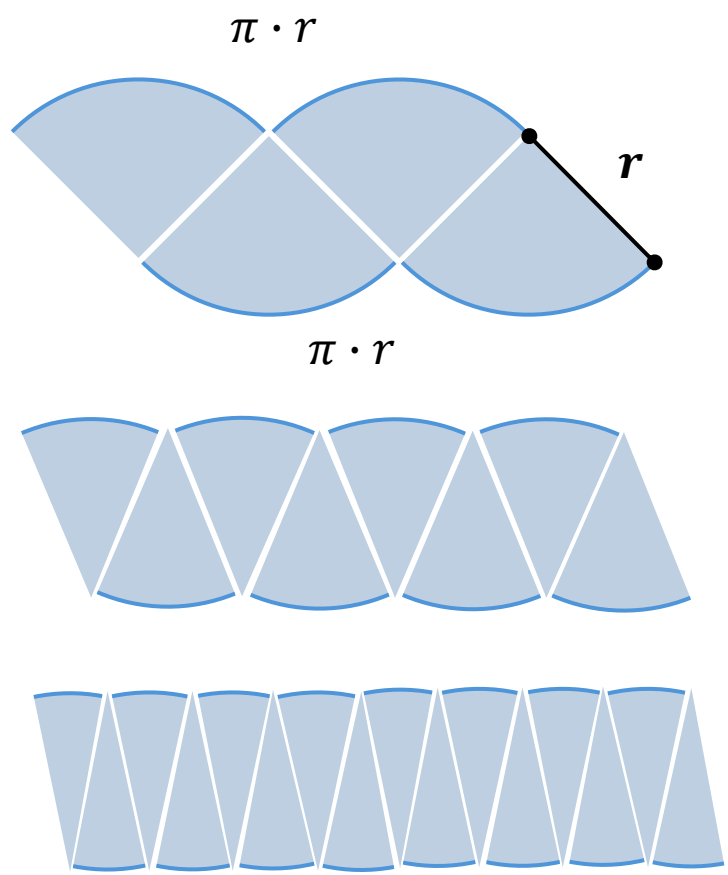
$$P = 2 \cdot \pi \cdot r$$

Semi – Perimeter:

$$P = \frac{\cancel{2} \cdot \pi \cdot r}{\cancel{2}} = \pi \cdot r$$



Performing succesive equal area sector partitions and rearranging:



Area Parallelogram:

$$A_{▱} = b \cdot h = \boldsymbol{\pi \cdot r^2} = \boldsymbol{A}_{\odot}$$

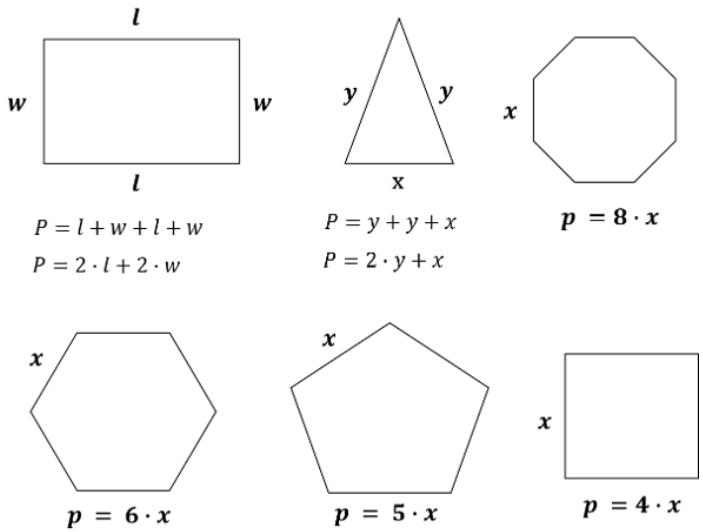
$h = \boldsymbol{r}$

$b = \boldsymbol{\pi \cdot r}$

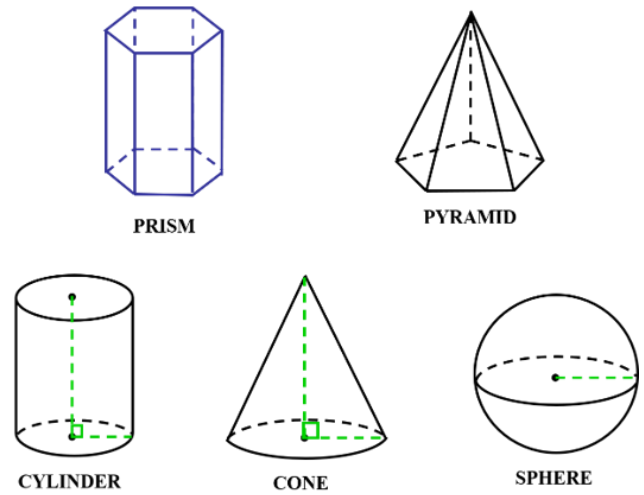
The limit of generating progressive partitions in the circle of equal área sectors is a parallelogram.

Solids: Surface Area and Volume

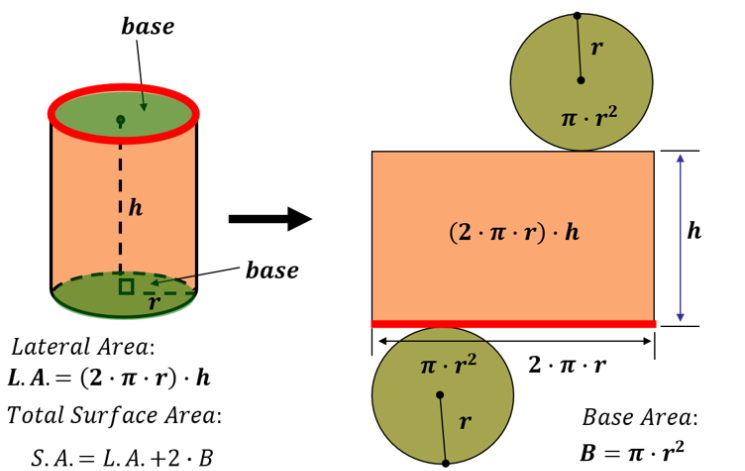
Perimeters Review



Solids



Surface Area in Cylinders



Lateral Area:

$L.A. = (2 \cdot \pi \cdot r) \cdot h$

Total Surface Area:

$S.A. = L.A. + 2 \cdot B$

$S.A. = (2 \cdot \pi \cdot r) \cdot h + 2 \cdot \pi \cdot r^2$

Base Area:

$B = \pi \cdot r^2$

$r = radius$

$h = height\ or\ Altitude$

Volume in Cylinders

$V = Volume$

$V = Bh$

$V = (\pi \cdot r^2) \cdot h$

Base Area:

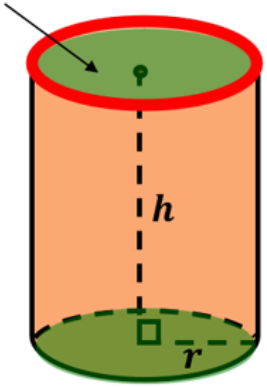
$B = \pi \cdot r^2$

$r = radius$

$h = height\ or\ Altitude$

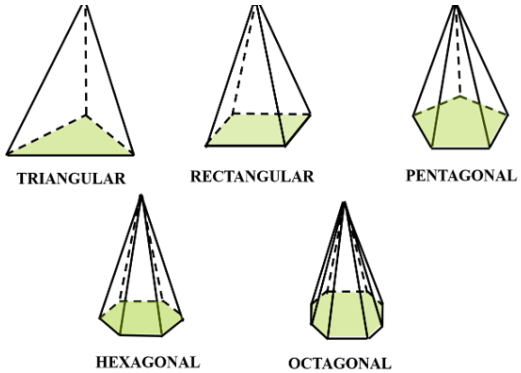
RIGHT CYLINDER

$B = \pi \cdot r^2$

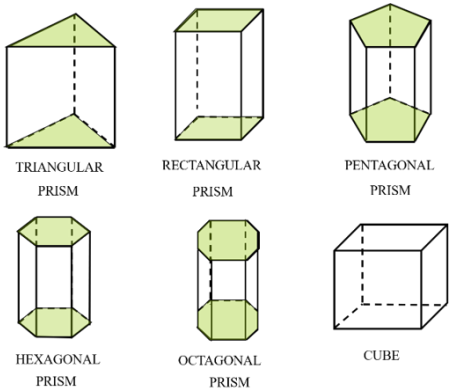


Solids: Surface Area and Volume

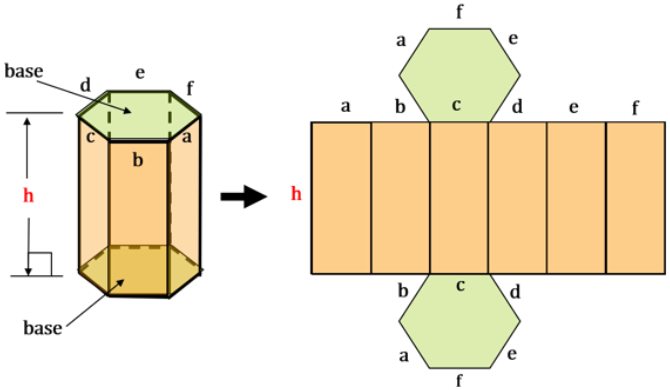
Tpes of Pyramids



Types of Prisms



Surface Area in Prisms



Lateral Area:

$L = ah + bh + ch + dh + eh + fh$

$L = (a + b + c + d + e + f)h$

Perimeter of base:

$P = a + b + c + d + e + f$

Substituting

$L = Ph$

Total Area= Lateral Area + 2 Base Area

$$T = L + 2B$$
$$T = Ph + 2B$$

Where:

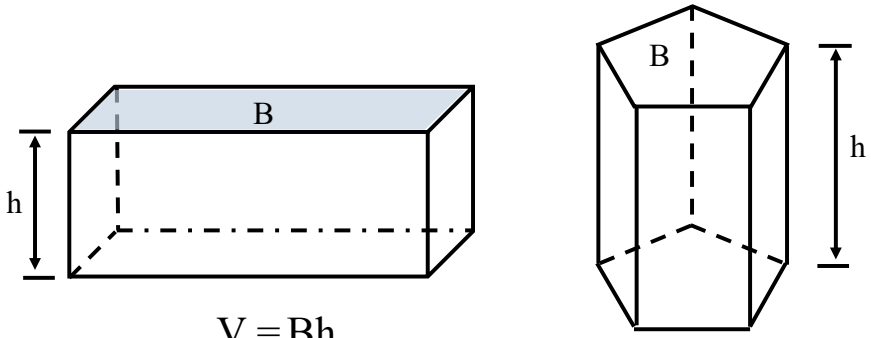
P=perimeter

h=height

B= base area

L= lateral area

Volume in a Right Prism



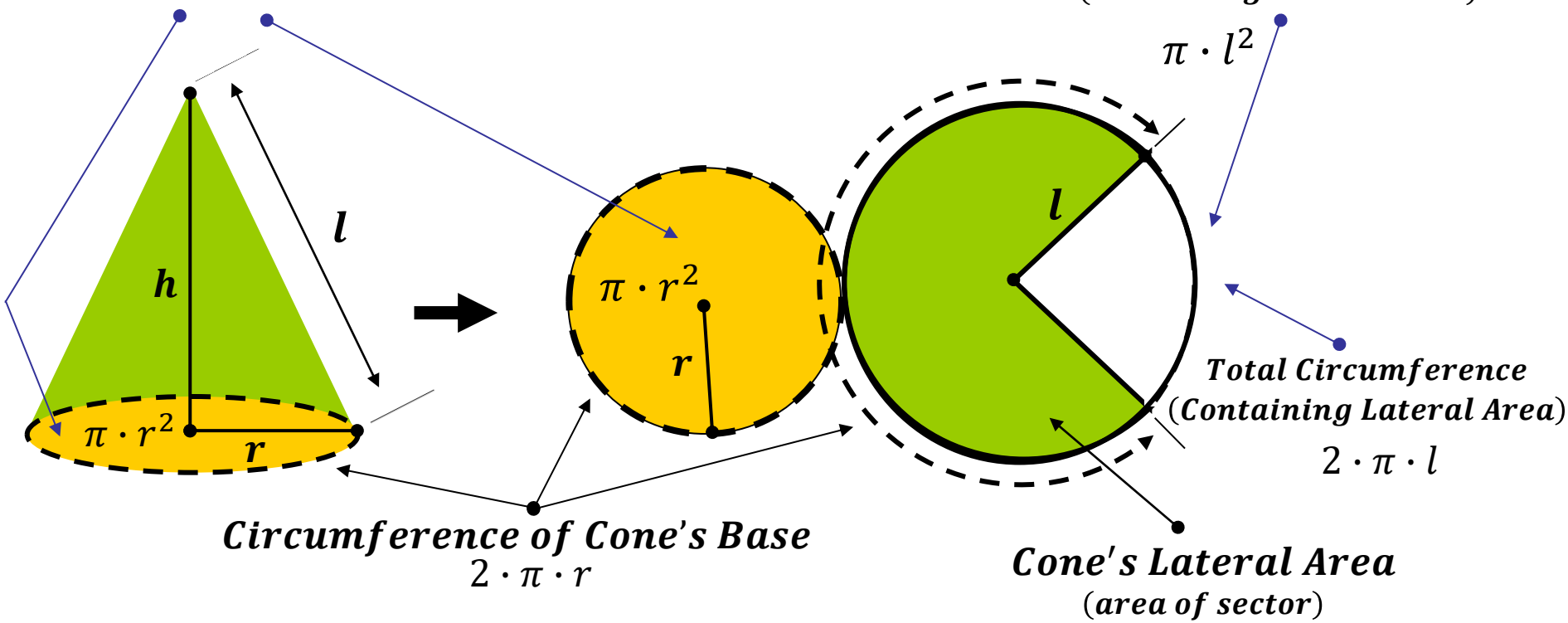
$V = Bh$

where:

B= Area of the base

h= height

Cone's Surface Area:



$$\frac{\text{Cone's Lateral Area}}{\text{Total Area Circle (Containing Lateral Area)}} = \frac{\text{Cone's Base Circumference}}{\text{Total Circumference (Containing Lateral Area)}}$$

The proportion to start the proof uses the circle containing the lateral area

$$\frac{\text{Cone's Lateral Area}}{\pi \cdot \cancel{l^2}} = \frac{2 \cdot \cancel{\pi} \cdot r}{2 \cdot \cancel{\pi} \cdot \cancel{l}}$$

$$\frac{\text{Cone's Lateral Area}}{\pi \cdot l} = r$$

$$\text{Cone's Lateral Area} = \pi \cdot r \cdot l$$

Total Surface Area:

$$S.A. = \text{Circle Base Area} + \text{Lateral Area}$$

$$S.A. = \pi \cdot r^2 + \pi \cdot r \cdot l$$

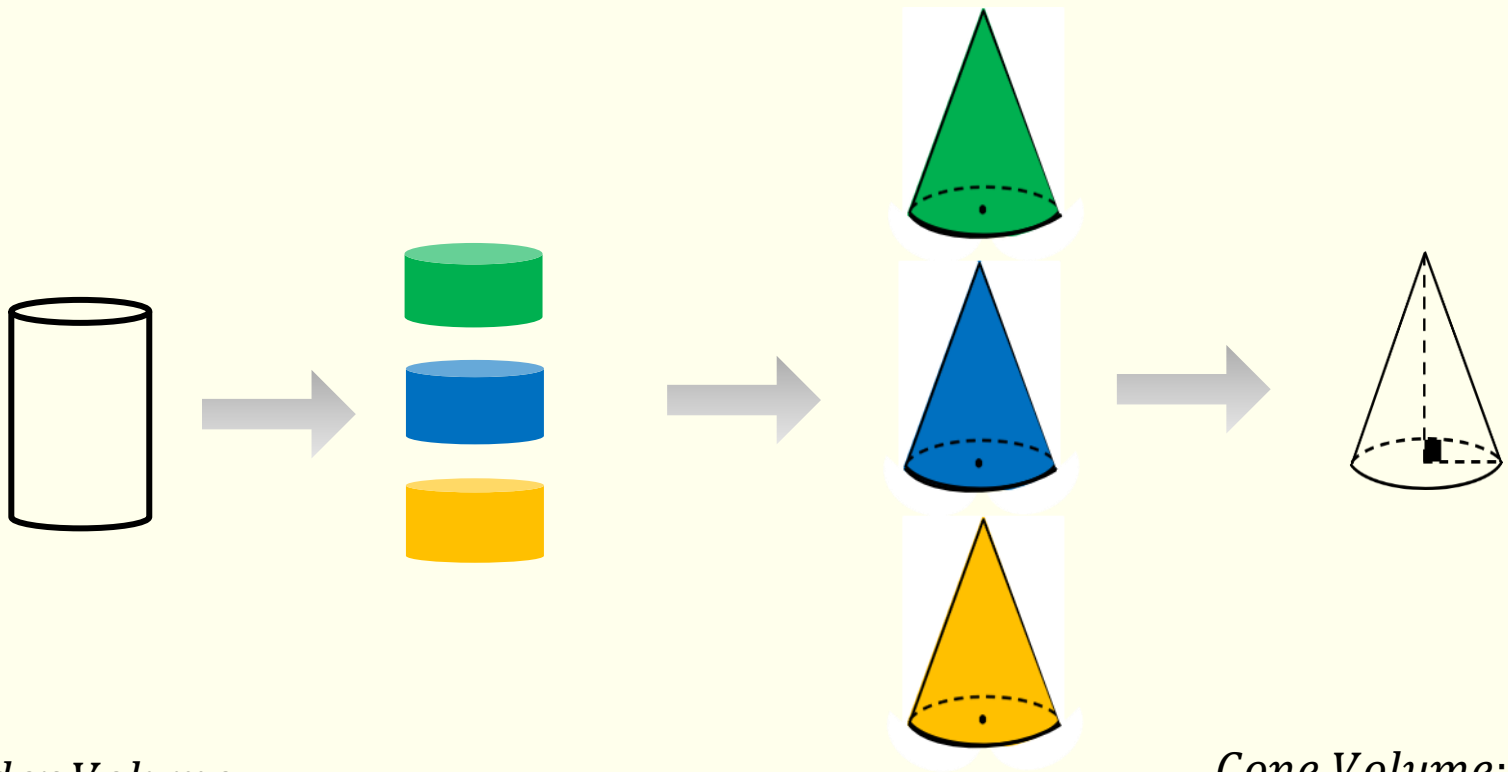
$r = \text{Radius}$

$B = \text{Circle Base Area}$

$l = \text{Slant Height}$

$h = \text{Height or Altitude}$

Comparing volume of a cylinder and a cone with the congruent bases and same height:



Cylinder Volume:

$$\boldsymbol{V = B \cdot h}$$

Where:

$V = Volume$

$B = Area\ Circle\ Base$

$h = height$

Volume of a cone fits three times in the volume of a cylinder with congruent circles at their bases and same height.

Cone Volume:

$$\boldsymbol{V = \frac{1}{3} \cdot B \cdot h}$$

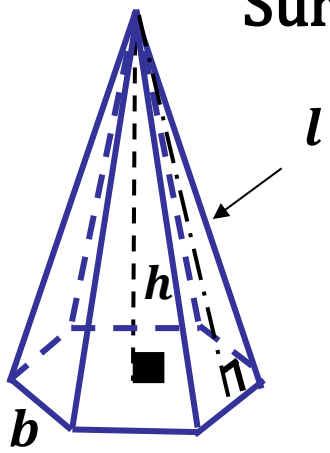
Where:

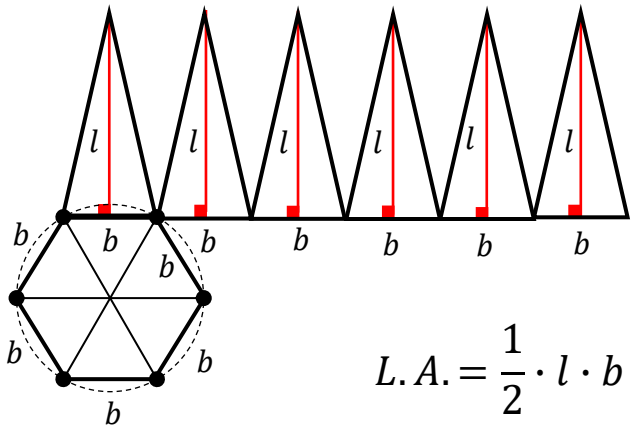
$V = Volume$

$B = Area\ Circle\ Base$

$h = height$

Surface Area in Pyramids





Lateral Area made up of Isosceles Δ's

$$Perimeter_{hexagon\ (convex\ polygon)}$$

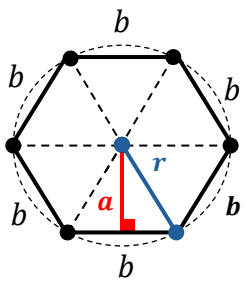
$$P = b + b + b + b + b + b$$

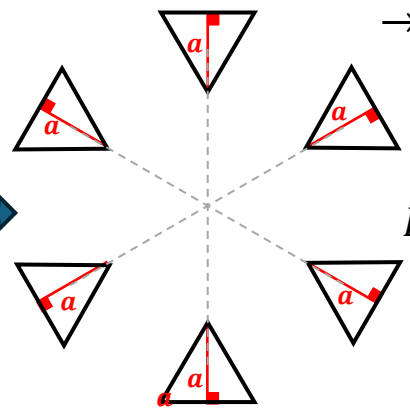
Lateral Area $_{pyramid}$:

$$L.A. = \frac{1}{2}\cdot l\cdot b + \frac{1}{2}\cdot l\cdot b + \frac{1}{2}\cdot l\cdot b + \frac{1}{2}\cdot l\cdot b + \frac{1}{2}\cdot l\cdot b + \frac{1}{2}\cdot l\cdot b$$

$$\rightarrow\ L.A. = \frac{1}{2}\cdot l\cdot(b+b+b+b+b+b) = \frac{1}{2}\cdot P\cdot l$$

$r = Radius$





$a = Apothem$

Base Area $_{hexagon}$:

$$B = \frac{1}{2}\cdot a\cdot b + \frac{1}{2}\cdot a\cdot b + \frac{1}{2}\cdot a\cdot b + \frac{1}{2}\cdot a\cdot b + \frac{1}{2}\cdot a\cdot b + \frac{1}{2}\cdot a\cdot b$$

$$\rightarrow\ B = \frac{1}{2}\cdot a\cdot(b+b+b+b+b+b)$$

$$B = \frac{1}{2}\cdot P\cdot a$$

$$\mathrm{L.}A. = \frac{1}{2}\cdot Perimeter\cdot Slant\ height$$

$$L.A. = \frac{1}{2}\cdot P\cdot l$$

$B = Base\ Area$

$l = Slant - height$

$h = height\ or\ Altitude$

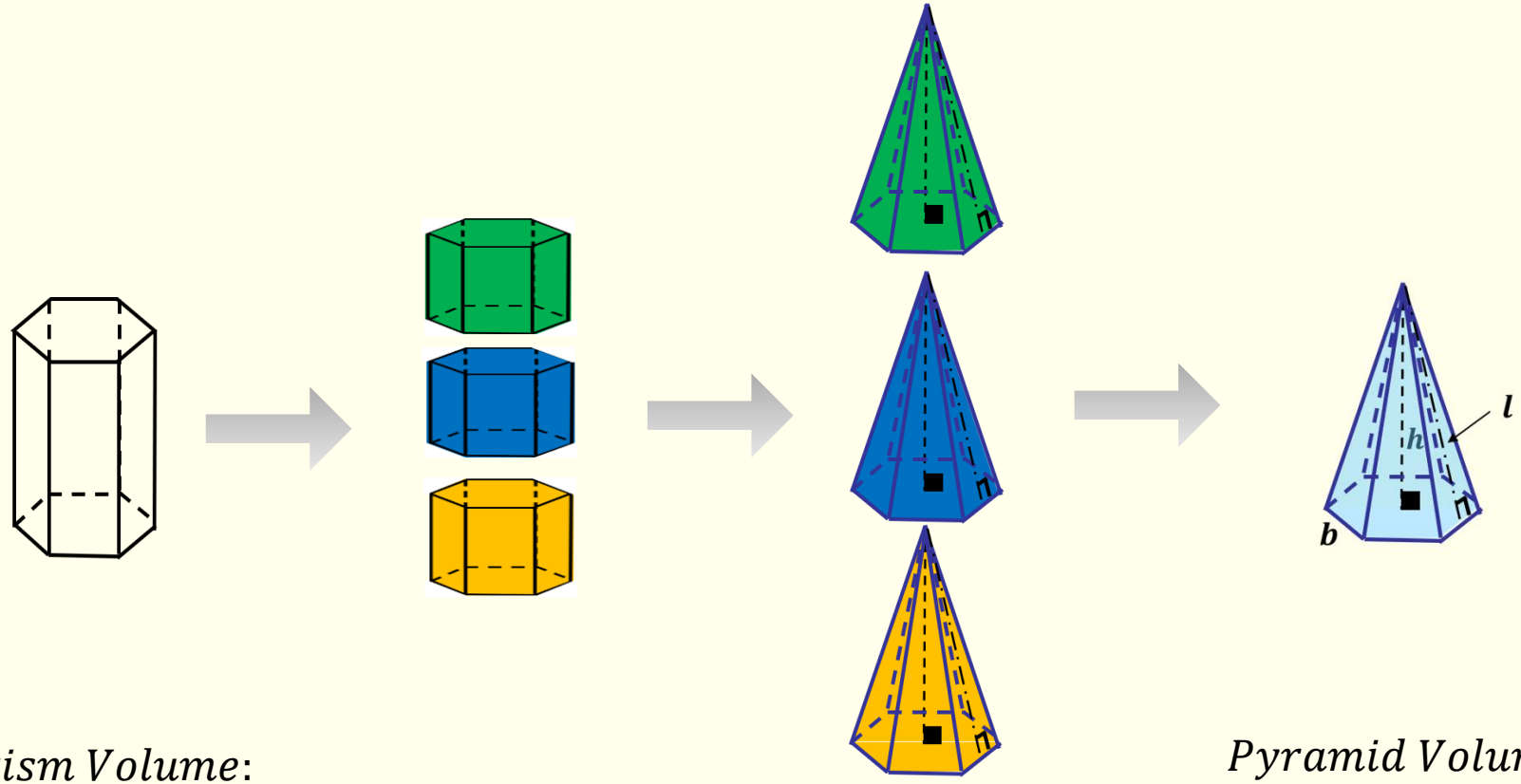
All equilateral Δ's are isosceles but not all isosceles Δ's are equilateral Δ's .

Total Surface Area $_{pyramid}$:

$$S.A. = Lateral\ Area + Base\ Area$$

$$S.A. = \frac{1}{2}\cdot P\cdot l + \frac{1}{2}\cdot P\cdot a$$

Comparing volume of a prism and a pyramid with the congruent bases and same height:



Prism Volume:

$$\boldsymbol{V = B \cdot h}$$

Where:

$V = Volume$

$B = Area\ Base\ Polygon$

$h = height$

Volume of a pyramid fits three times in the volume of a prism with congruent polygons at their bases and same height.

Pyramid Volume:

$$\boldsymbol{V = \frac{1}{3} \cdot B \cdot h}$$

Where:

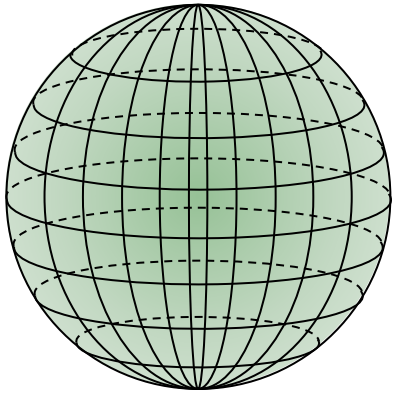
$V = Volume$

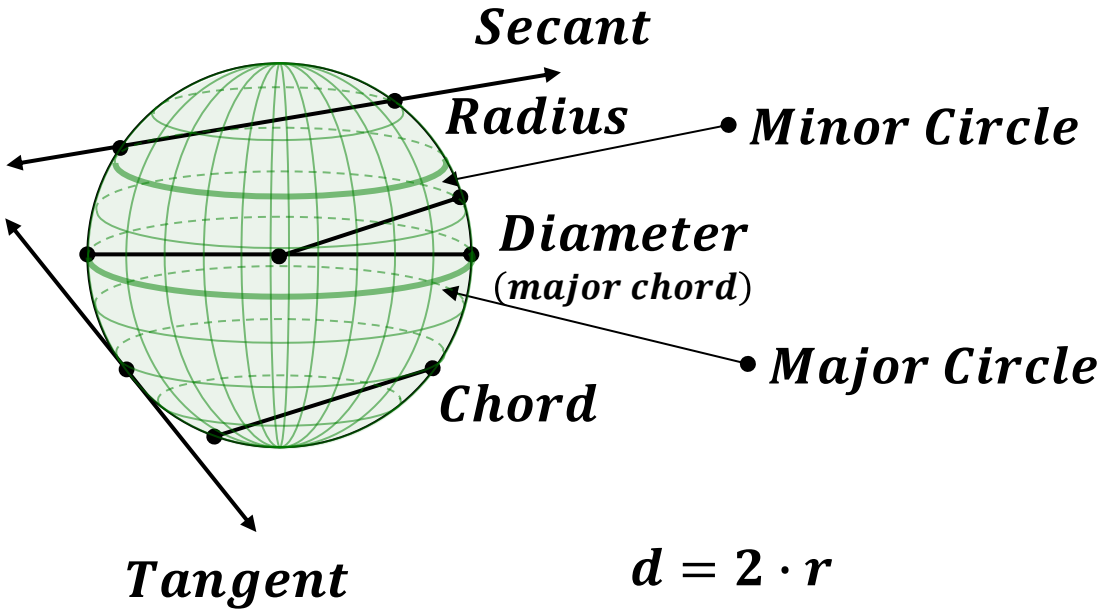
$B = Area\ Base\ Polygon$

$h = height$

SPHERE FEATURES

SPHERE

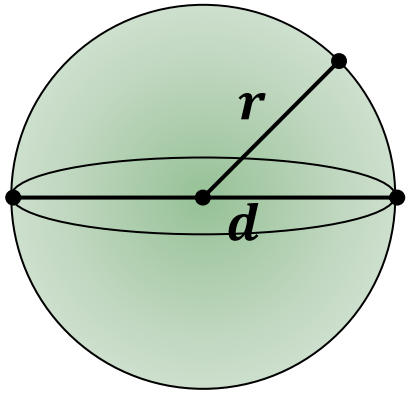




$$d = 2 \cdot r$$

or

$$r = \frac{1}{2} \cdot d$$



Sphere Surface Area:

$$S.A. = 4 \cdot \pi \cdot r^2$$

Sphere Volume:

$$V = \frac{4}{3} \cdot \pi \cdot r^3$$

Where:

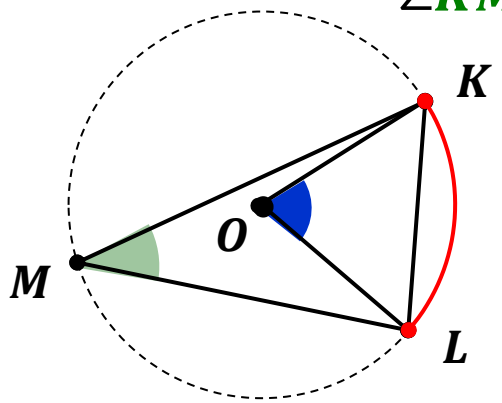
$S.A. =$ *Surface Area*

$V =$ *Volume*

$r =$ *Radius*

$d =$ *Diameter*

Inscribed Angle vs Central Angle and Arc vs Subtended Chord to Arc

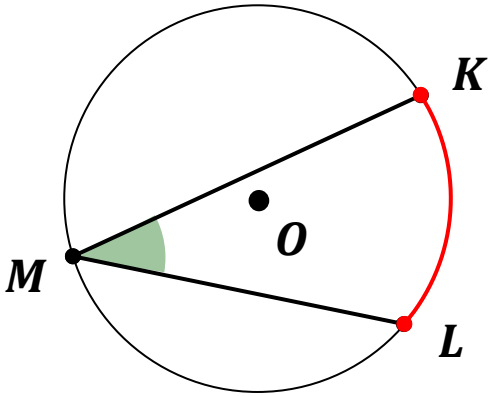


$$\angle KML \rightarrow \textit{Inscribed Angle} = \begin{cases} \textit{Vertex is on the } \odot \\ \textit{Rays are chords in } \odot \end{cases}$$

$$\widehat{KL} \rightarrow \textit{Intersected arc with subtended chord } \overline{KL}$$

$$\angle KOL \rightarrow \textit{Central Angle} = \begin{cases} \textit{Vertex on center of } \odot \\ \textit{Rays are Radii in } \odot \end{cases}$$

$$m\angle KOL = m\widehat{KL} \rightarrow \left\{ \begin{matrix} \textit{measurement of central } \angle \textit{ is equal to} \\ \textit{measurement of interseced arc} \end{matrix} \right\}$$



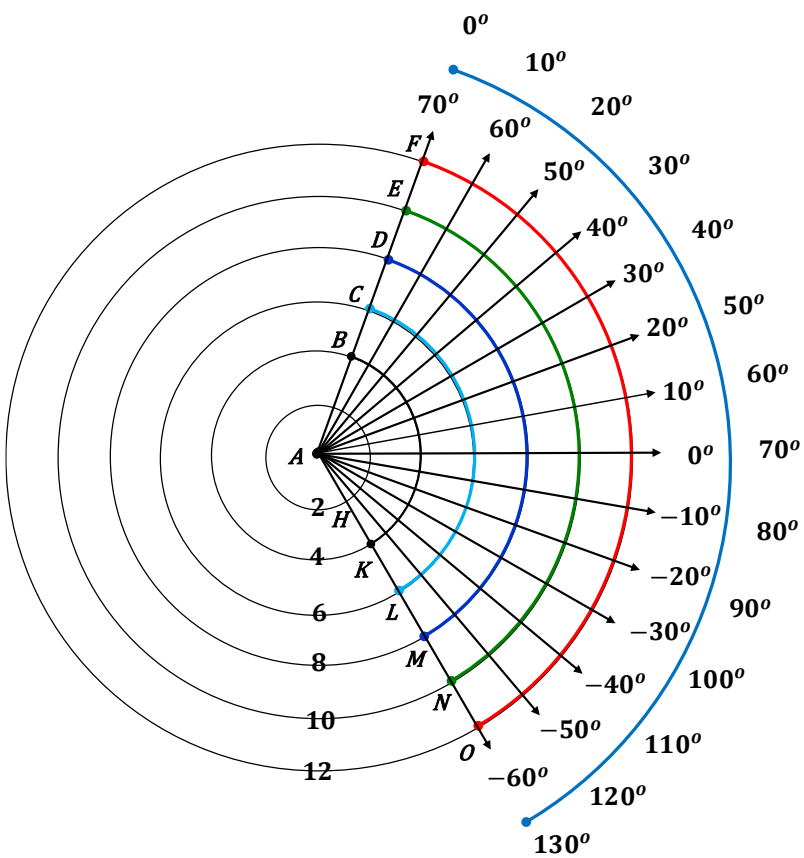
$$\textit{If}\{\textit{an angle is inscribed in a circle}\}$$

$$\textit{then} \left\{ \begin{matrix} \textit{the measure of the angle equals} \\ \textit{one} - \textit{half the measure of its intercepted arc.} \end{matrix} \right\}$$

$$m\angle KLM = \frac{1}{2} m\widehat{KL}$$

Arc's Length vs Arc's Angular measurement

$$\textbf{Arc's Length} = 2 \cdot \pi \cdot r \cdot \left(\frac{\theta^{O}}{360^{O}}\right) \rightarrow \left\{\begin{array}{c}\textbf{Arc's length is determined by the proportion} \\ \textbf{of its angular measurement respect} \\ \textbf{the total degrees in the circumference.} \\ \textbf{Angular units may be in Degrees or Radians}\end{array}\right\}$$



$m\widehat{FO} = 130^{O} \rightarrow$ *Angular measurement for* $\widehat{FO}$

$FO = 2 \cdot \pi \cdot \mathbf{12} \cdot \left(\frac{130^{O}}{360^{O}}\right) \rightarrow$ *linear units for* $\widehat{FO}$

$m\widehat{EN} = 130^{O} \rightarrow$ *Angular measurement for* $\widehat{EN}$

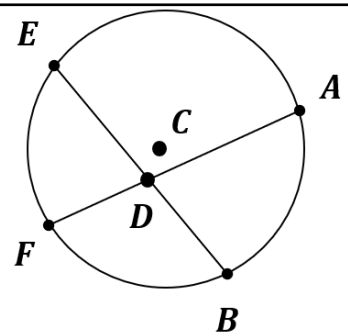
$EN = 2 \cdot \pi \cdot \mathbf{10} \cdot \left(\frac{130^{O}}{360^{O}}\right) \rightarrow$ *linear units for* $\widehat{EN}$

$m\widehat{DM} = 130^{O} \rightarrow$ *Angular measurement for* $\widehat{DM}$

$\mathrm{DM} = 2 \cdot \pi \cdot \mathbf{8} \cdot \left(\frac{130^{O}}{360^{O}}\right) \rightarrow$ *linear units for* $\widehat{DM}$

Circles: Arcs, Chords, Secants and Tangents

Two Chords Intersecting in a Circle

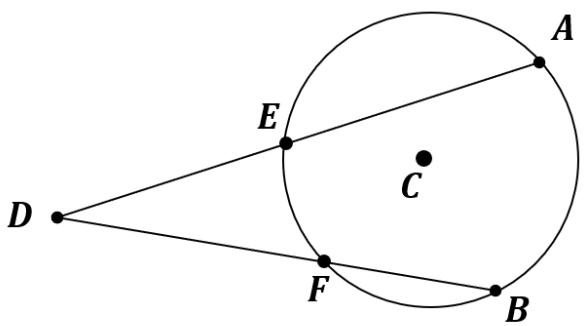


$$ED \cdot DB = FD \cdot AD$$

If {Two chords intersect in a circle}

then $\left\{\begin{matrix}\text{The product of their corresponding segments} \\ \text{formed by the intersection are equal}\end{matrix}\right\}$

Two Secant Intersecting in a Point at the Exterior of a Circle



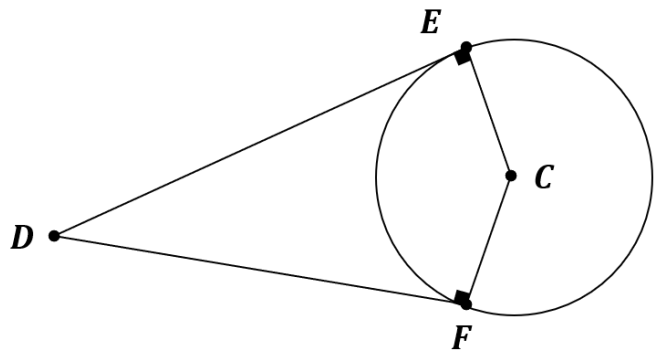
$$AD \cdot DE = BD \cdot BF$$

If {Two secants intersect in a point at the exterior of a circle}

then $\left\{\begin{matrix}\text{The product of the secant's length by the length of its exteror} \\ \text{correspondintg segment is equal to the product of the other secant's} \\ \text{length by its corresponding exterior segment length}\end{matrix}\right\}$

Circles: Arcs, Chords, Secants and Tangents

Two Tangets Intersecting in a Point at the Exterior of a Circle



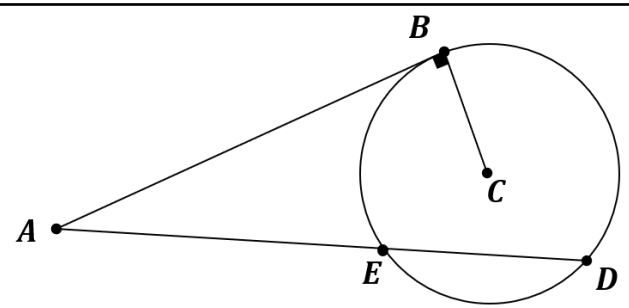
$$\overline{ED} \cong \overline{FD}$$

$$\left\{ \begin{array}{c} \overline{ED} \perp \overline{EC} \\ and \\ \overline{FD} \perp \overline{FC} \end{array} \right\}$$

If {*Two tangents intersect in a point at the exterior of a circle*}

then $\left\{ \begin{array}{c} \text{They are} \cong \text{and they are} \perp \text{at the point of} \\ \text{tangency with their corresponding radii} \end{array} \right\}$

A Tanget and a Secant Intersecting in a Point at the Exterior of a Circle



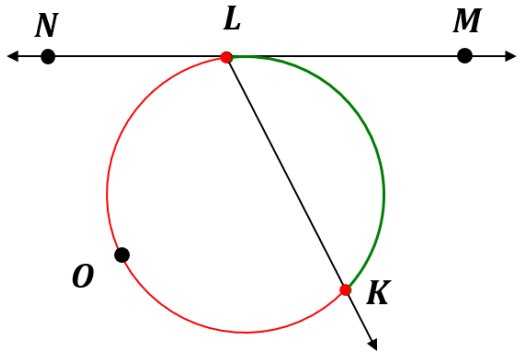
$$AB^2 = AD \cdot AE$$

If $\left\{ \begin{array}{c} \text{A tangent and a secant intersect in} \\ \text{a point at the exterior of a circle} \end{array} \right\}$

then $\left\{ \begin{array}{c} \text{The square of the tangent's length is equal to} \\ \text{the product of the secant's full length by its exterior} \\ \text{segment's length} \end{array} \right\}$

Circles: Arcs, Chords, Secants and Tangents

A Secant and a Tangent Intersect at the Point of Tangency



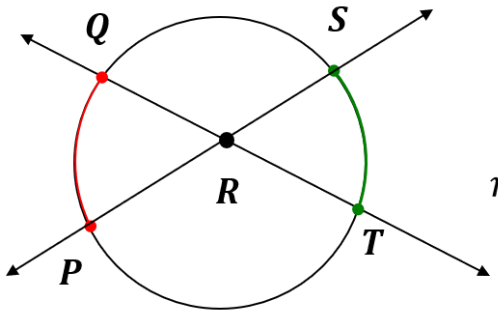
$$m\angle KLM = \frac{1}{2} m\widehat{KL}$$

$$m\angle KLN = \frac{1}{2} m\widehat{KOL}$$

If {a secant and a tangent intersect at the point of tangency}

then $\left\{ \begin{array}{c} \text{the measure of each angle formed is} \\ \text{one} - \text{half the measure of its intercepted arc.} \end{array} \right\}$

Two Secants Intersect in the Interior of a Circle



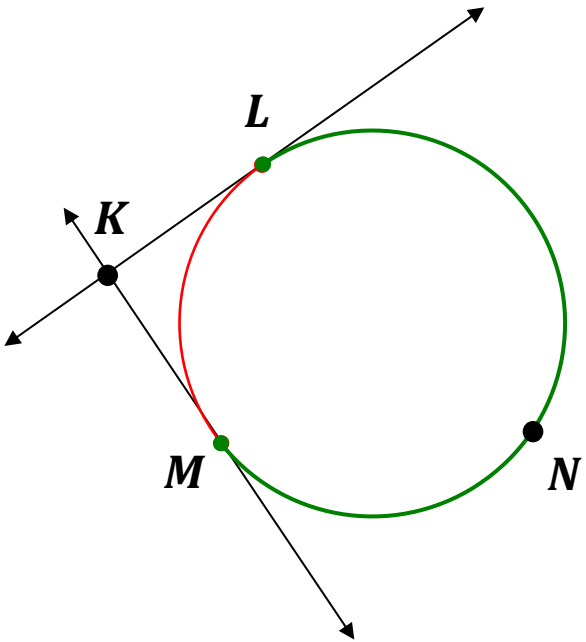
$$m\angle QRP = \frac{1}{2}\left(m\widehat{ST} + m\widehat{QP}\right)$$

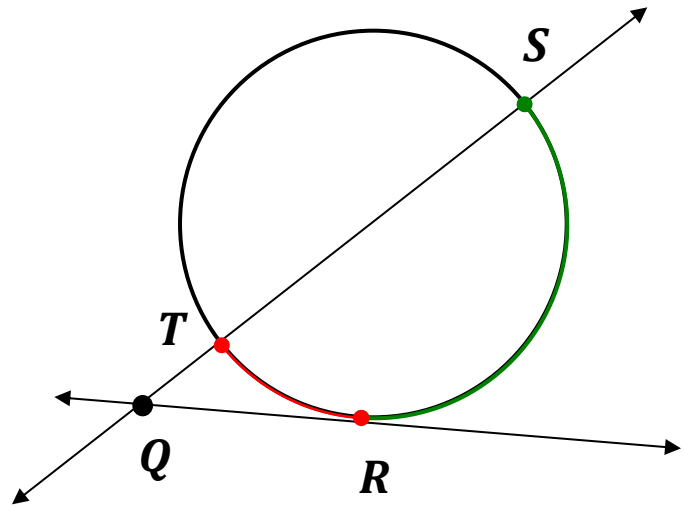
If {two secants intersect in the interior of a circle}

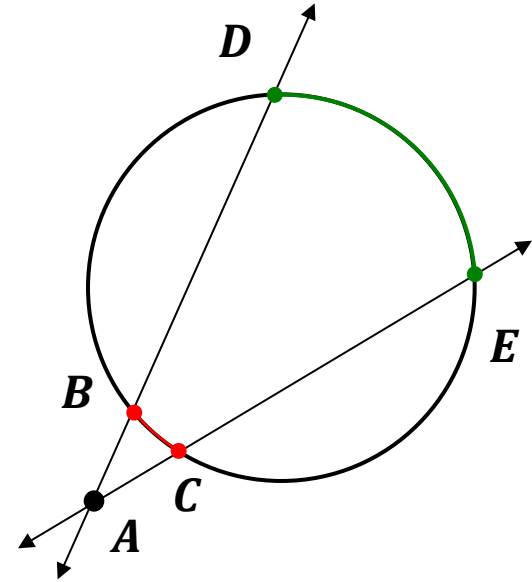
then $\left\{ \begin{array}{c} \text{the measure of each angle formed is} \\ \text{one} - \text{half the sum of the mearures of the arcs} \\ \text{intercepted by the angle and ists vertical angle.} \end{array} \right\}$

Secant and a tangent

$$m\angle TQR = \frac{1}{2}\left|m\widehat{SR} - m\widehat{TR}\right|$$







Two tangents

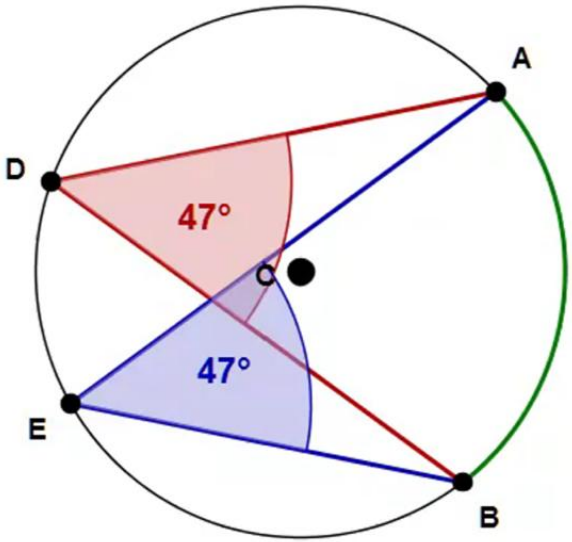
$$m\angle LKM = \frac{1}{2}\left|m\widehat{LNM} - m\widehat{LM}\right|$$

Two secants

$$m\angle BAC = \frac{1}{2}\left|m\widehat{DE} - m\widehat{BC}\right|$$

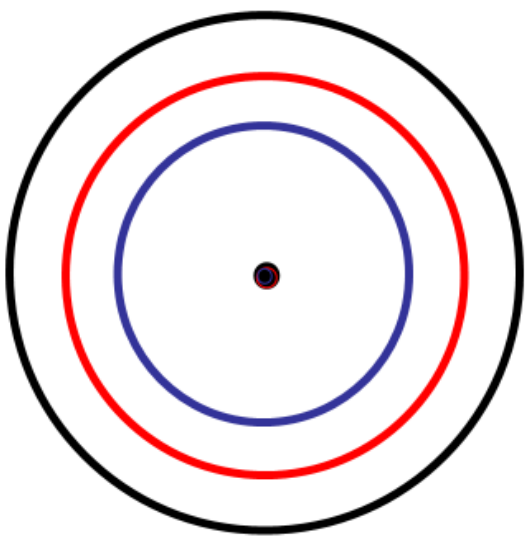
$$\text{If}\left\{\begin{matrix}\text{two secants, a secant and a tangent, or two tangents}\\ \text{intersect in the exterior of a circle}\end{matrix}\right\}$$

$$\text{then}\left\{\begin{matrix}\text{the measure of the formed angle is}\\ \text{the absolute value of one} - \text{half the}\\ \text{difference of the measures of the intercepted}\\ \text{arcs or their positive semi} - \text{difference.}\end{matrix}\right\}$$



$$\boldsymbol{If\ \{\angle ADB\ and\ \angle BEA\ \ intercept\ \ \widehat{AB}\}}$$
$$\boldsymbol{then\{\angle ADB\ \cong \angle BEA\}}$$

$$\boldsymbol{If\left\{\begin{array}{c}\text{Two inscribed angles of a circle or congruent}\\ \text{circles intercept congruent arcs, or the same arc}\end{array}\right\}}$$
$$\boldsymbol{then\ \{\ The\ angles\ are\ congruent\ \}}$$



A set of $\odot'$ *s sharing the same center are* ***CONCENTRIC*** *and all* $\odot'$ *s are* $\sim$ *since they have same shape and equal size or different Sizes*

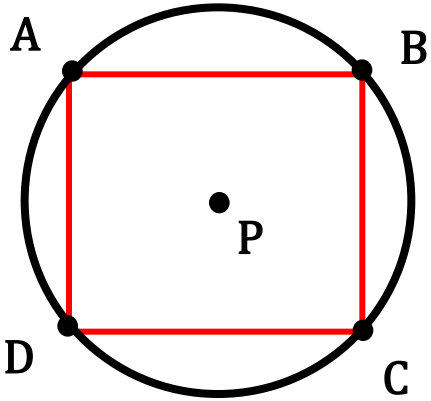
$$\boldsymbol{a:b}$$

where $\boldsymbol{a \neq b}$ *or* $a = b$ *and "*$\boldsymbol{a}$*" and "*$\boldsymbol{b}$*" are* ***radii***

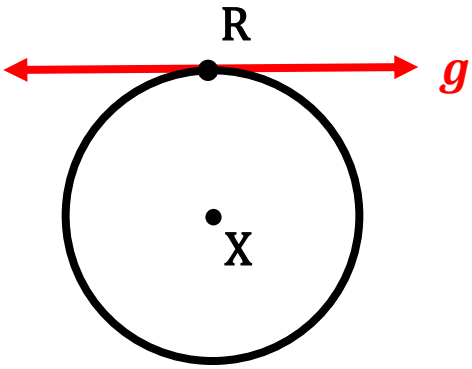
if $\{\boldsymbol{a = b}\}$ *then* $\{$***they are*** $\sim$ ***and*** $\cong\}$

if $\{\boldsymbol{a \neq b}\}$ *then* $\{$***they are only*** $\sim$***and*** $\ncong\}$

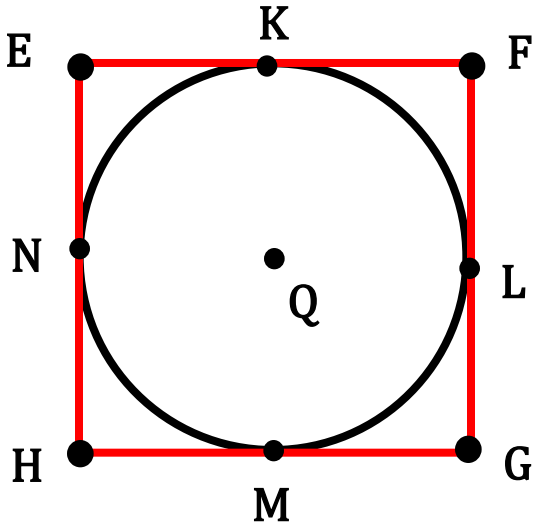
Inscribed vs Circumscribed



Quadrilateral ABCD is ***inscribed*** *to circle P.*



Line g is TANGENT to circle X at point R.



Quadrilateral EFGH is ***circumscribed*** *to circle Q, having sides to be TANGENT at points K, L, M and N.*

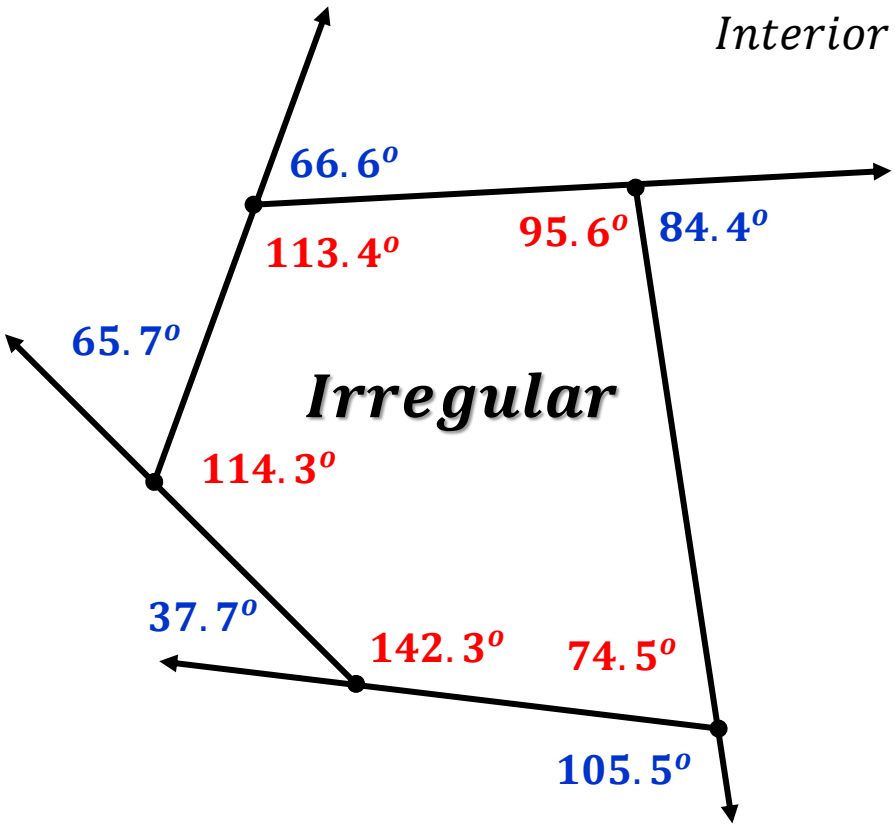
Irregular vs Regular
(convex polygons)

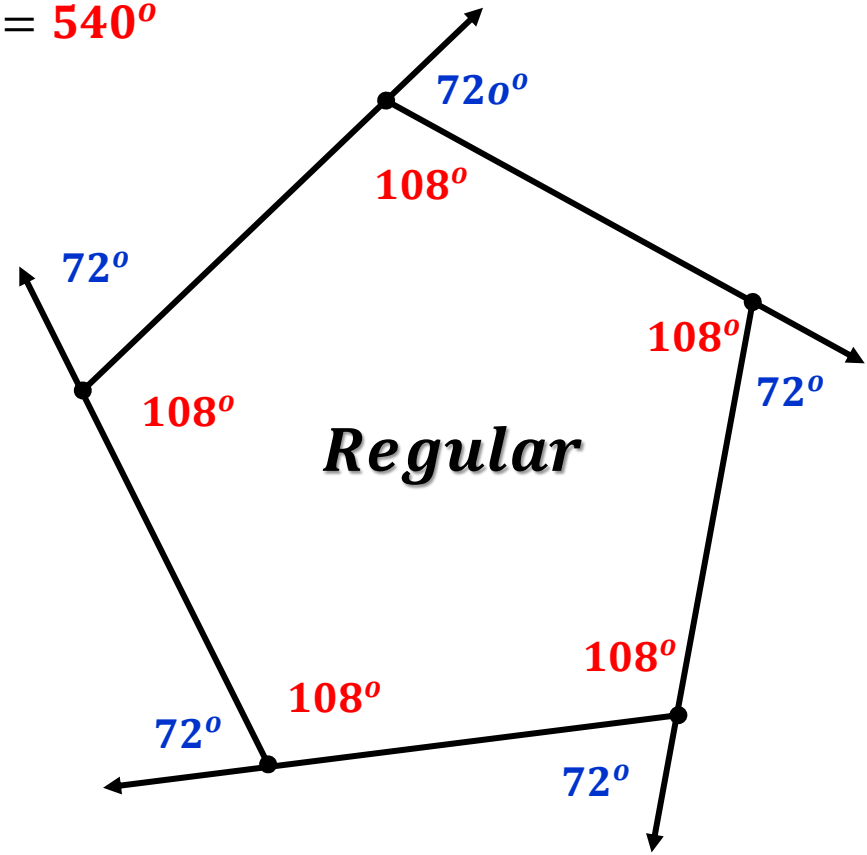
$Exterior\ Sum = 72^o + 72^o + 72^o + 72^o + 72^o$

$Exterior\ Sum = 360^o$

$Interior\ Sum = 108^o + 108^o + 108^o + 108^o + 108^o$

$Interior\ Sum = 540^o$





$Exterior\ Sum = 65.7^o + 66.6^o + 84.4^o + 105.5^o + 37.7^o$

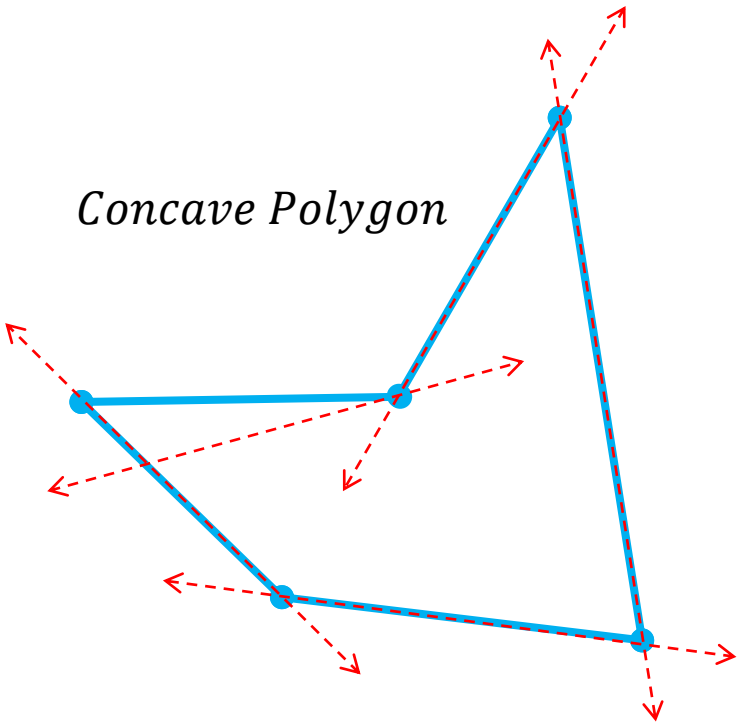
$Exterior\ Sum = 360^o$

$Interior\ Sum = 114.3^o + 113.4^o + 95.6^o + 74.5^o + 142.3^o$

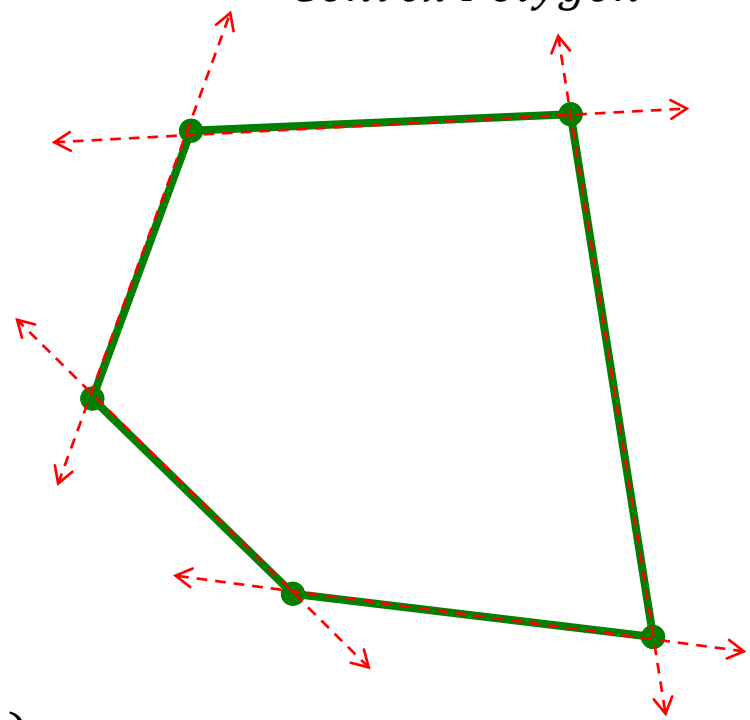
$Interior\ Sum = 540^o$

If $\left\{\begin{array}{c}\textit{Extending the sides of a polygon}\\ \textit{beyond the intersection endpoints}\\ \textit{of consecutive sides the } \textbf{extending lines}\\ \textbf{go through the interior of the polygon}\end{array}\right\}$ *then* {*It is a* ***Concave Polygon***}

Concave Polygon



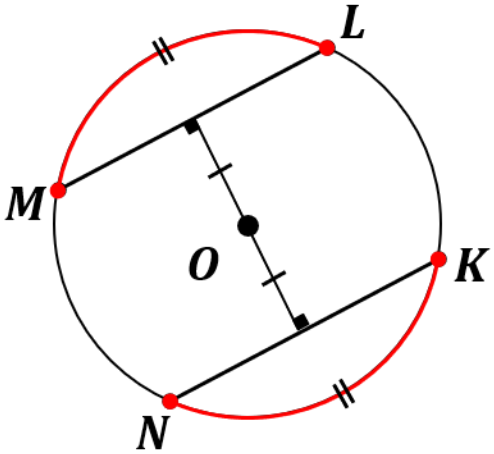
Convex Polygon



If $\left\{\begin{array}{c}\textit{Extending the sides of a polygon}\\ \textit{beyond the intersection endpoints}\\ \textit{of consecutive sides the } \textbf{extending lines}\\ \textbf{do not go through the interior of}\\ \textbf{the polygon}\end{array}\right\}$ *then* {*It is a* ***Convex Polygon***}

Circles: Arcs, Chords, Secants and Tangents

Theorem about arcs and Chordss and its converse



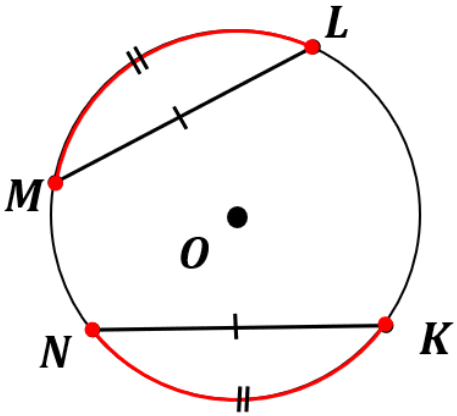
If {In a ⊙ or in ⊙ ≅′ s two arcs are ≅}

then {The arcs to which they are subtended are also ≅}

If {The arcs to which they are subtended are also ≅}

then {In the ⊙ or in ⊙ ≅′ s two chords are ≅}

Theorem about arcs and Chordss and its converse



If {In a ⊙ or in ⊙ ≅′ s two arcs are ≅}

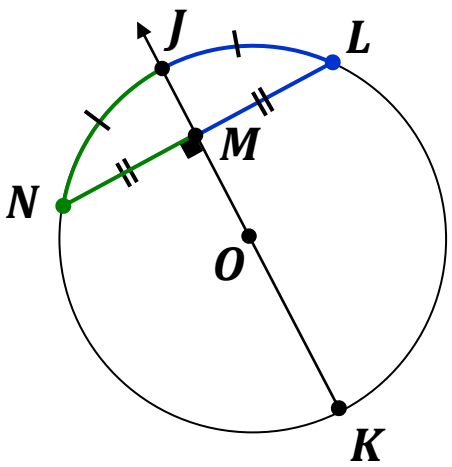
then {The subtended chords are also ≅}

Si{{In a ⊙ or in ⊙ ≅′ s two chords are ≅}}

entonces {The arcs to which they are subtended are also ≅}

if $\{\widehat{LM} \cong \widehat{NK}\}$ then $\{\overline{LM} \cong \overline{NK}\}$ or if $\{\overline{LM} \cong \overline{NK}\}$ then $\{\widehat{LM} \cong \widehat{NK}\}$

Theorem about a Diameter or Radius Intersecting a Chord in a Right Angle



If $\left\{\begin{matrix}\textbf{in a } \odot \textbf{ a chord is perpendicular} \\ \textbf{to a radius or diameter}\end{matrix}\right\}$

then $\left\{\begin{matrix}\textbf{both the chord and the arc to which} \\ \textbf{the chord is subtended are bisected by the} \\ \textbf{radius or diameter.}\end{matrix}\right\}$

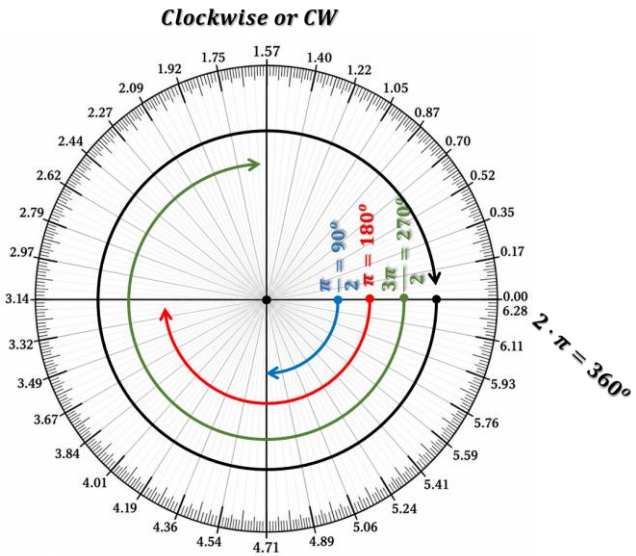
if $\left\{\begin{matrix}\textbf{both the chord and the arc to which} \\ \textbf{the chord is subtended are bisected by the} \\ \textbf{radius or diameter.}\end{matrix}\right\}$

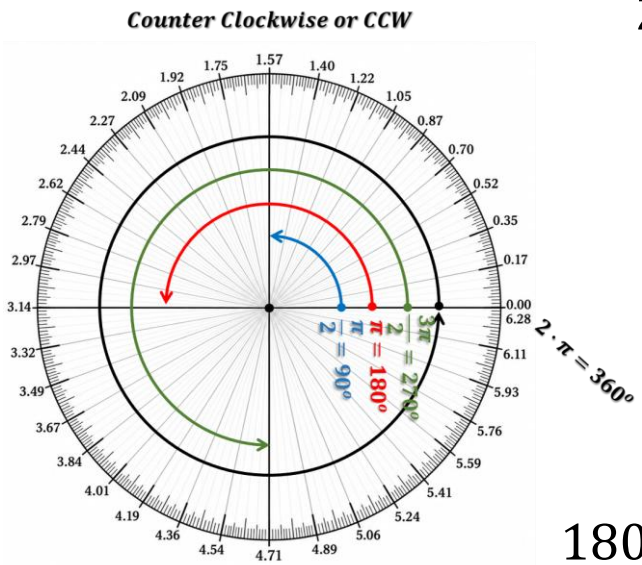
then $\left\{\begin{matrix}\textbf{in the } \odot \textbf{ the chord is perpendicular} \\ \textbf{to the radius or diameter}\end{matrix}\right\}$

$$if\ \{\overline{KJ}\ \perp \overline{LN}\}\ then\ \{\overline{NM} \cong \overline{LM}\ \ y\ \ \widehat{NJ} \cong \widehat{LJ}\}$$

$$o$$

$$if\ \{\overline{NM} \cong \overline{LM}\ \ y\ \ \widehat{NJ} \cong \widehat{LJ}\}\ then\ \{\{\overline{KJ}\ \perp \overline{LN}\}\}$$





$$2 \cdot 180^o = 2 \cdot \pi \; Radians$$

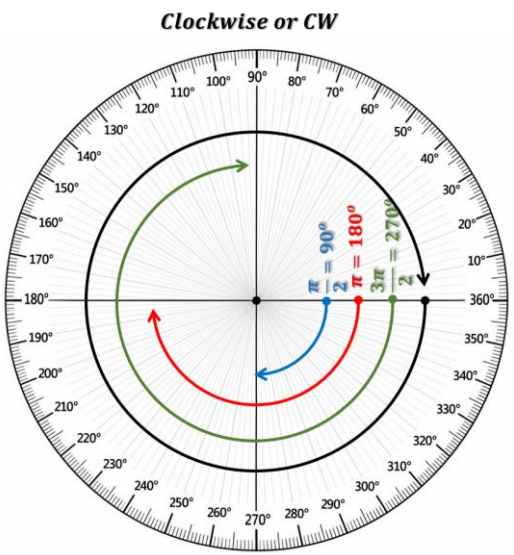
$$360^o = 2 \cdot \pi \; Radians$$

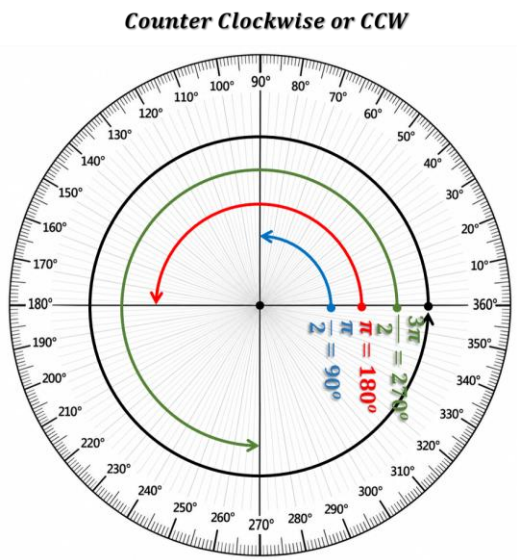
$$\frac{360^o}{2} = \frac{2 \cdot \pi \; Radians}{2}$$

$$180^o = \pi \; Radians$$

$$\frac{180^o}{180^o} = \frac{\pi}{180^o}$$

$$1^o = \frac{3.14}{180} \approx 0.018 \; Radians$$





$$180^o = \pi$$

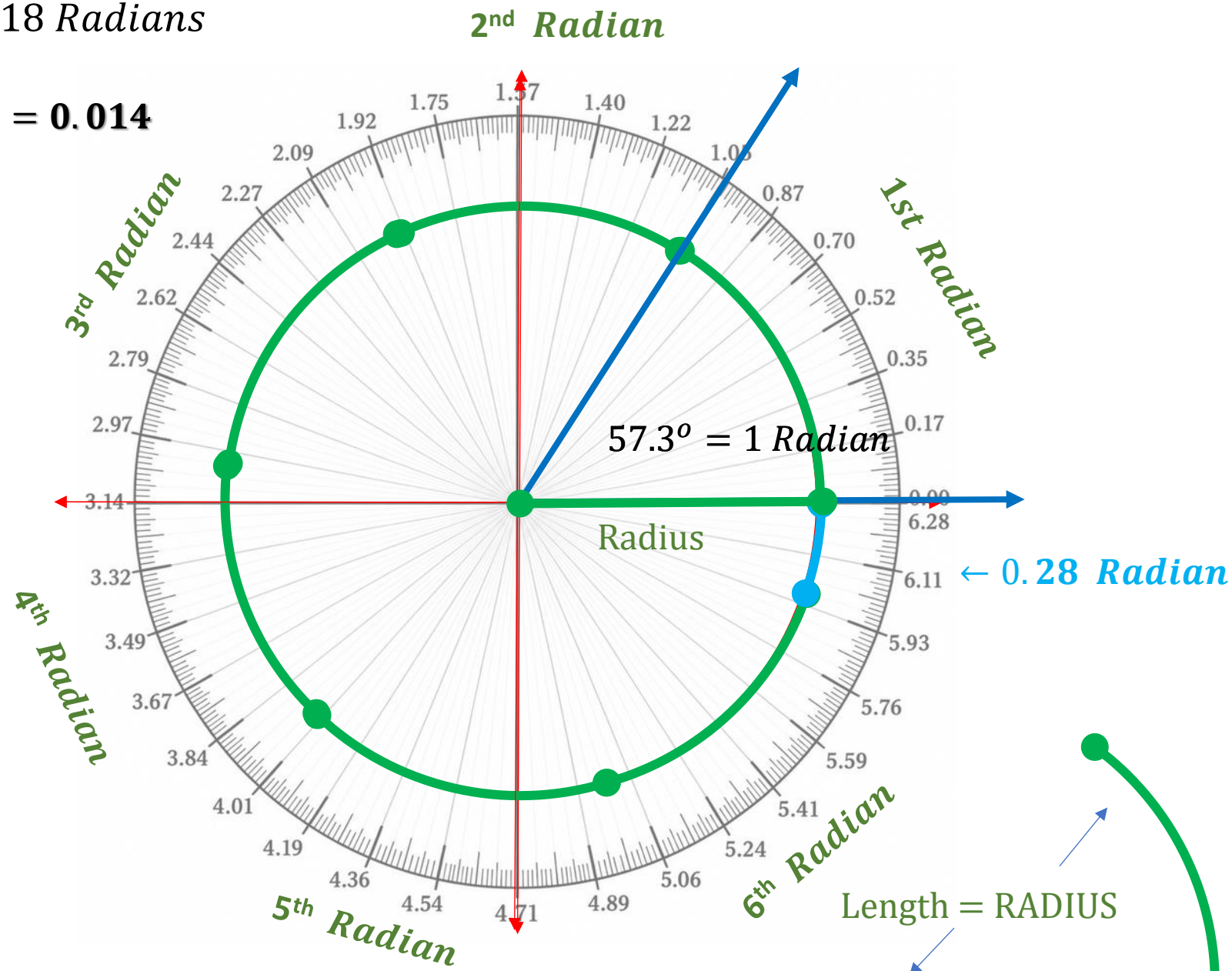
$$\frac{180^o}{\pi} = \frac{\pi}{\pi}$$

$$\frac{180^o}{3.14} = \frac{\pi}{\pi}$$

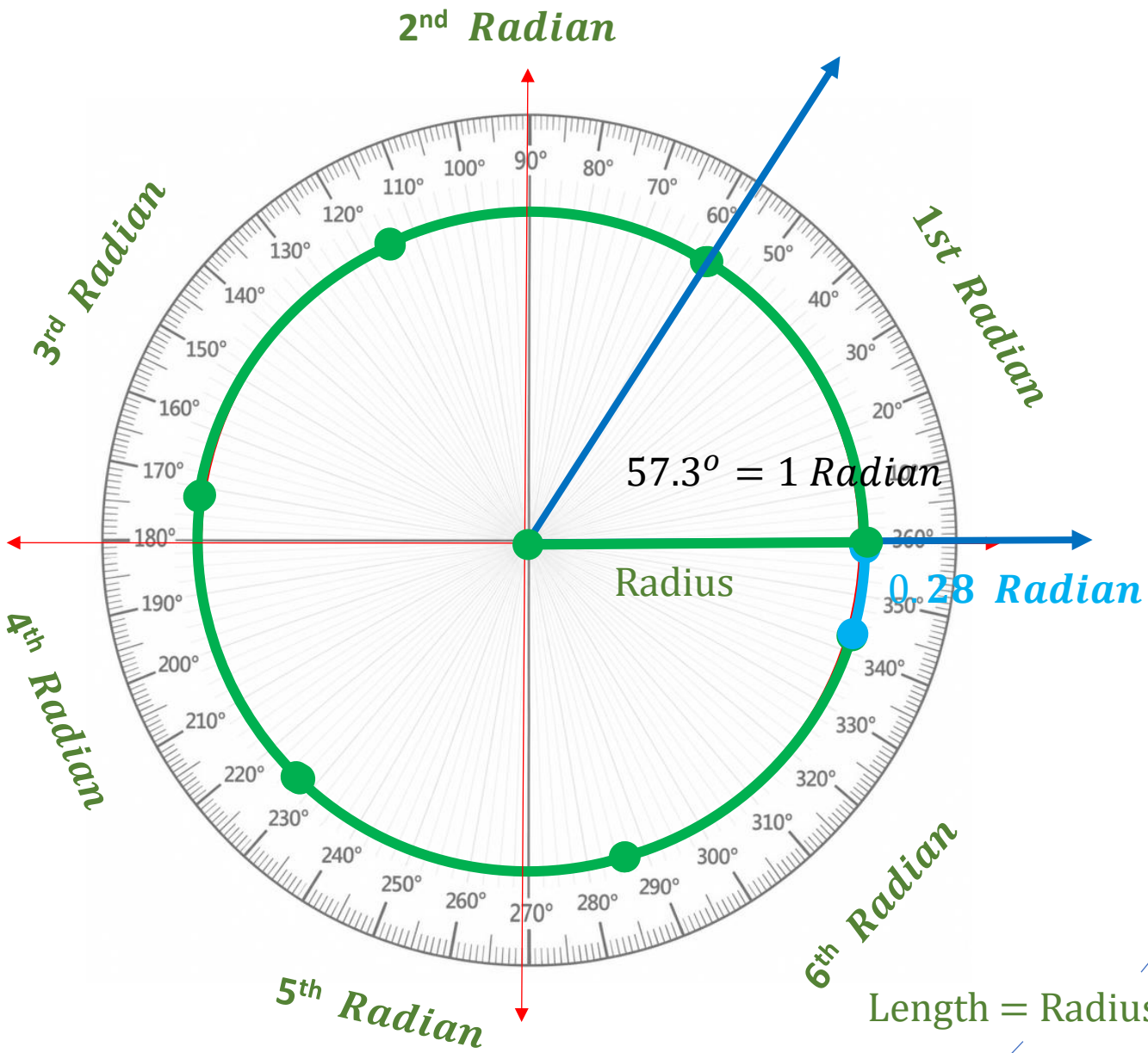
$$57.3^o = 1 \; Radian$$

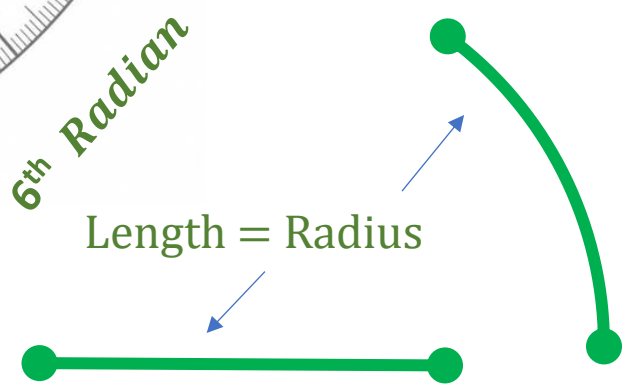
$1^o \approx 0.018\ Radians$

Each line $= \mathbf{0.014}$



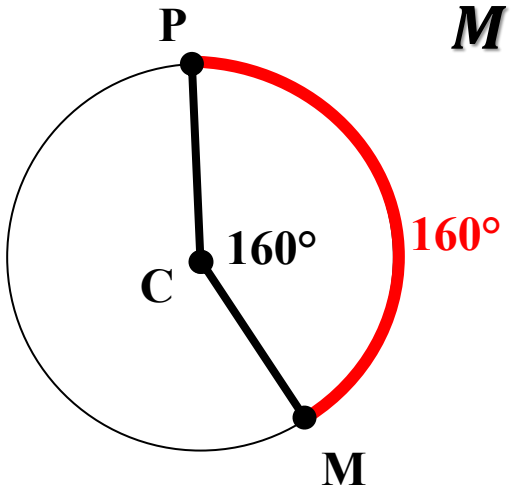
$\mathbf{2 \cdot \pi = 2 \cdot 3.14 = 6.28\ Radians}$





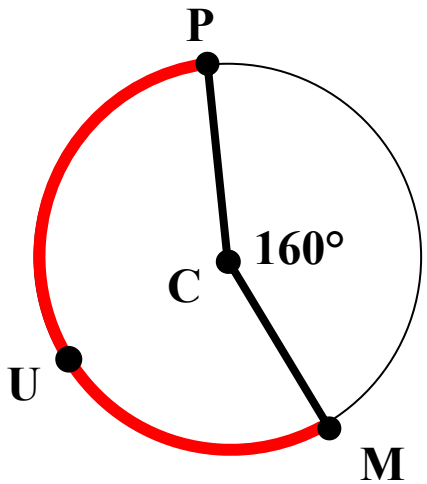
$$2 \cdot \pi = 2 \cdot 3.14 = 6.28\ Radians$$

Minor arcs, Major Arcs and Semi – Circle



The measure of a minor arc is the measure of its central angle.

$$\angle PCM = 160^o \rightarrow m\widehat{PM} = 160^o$$



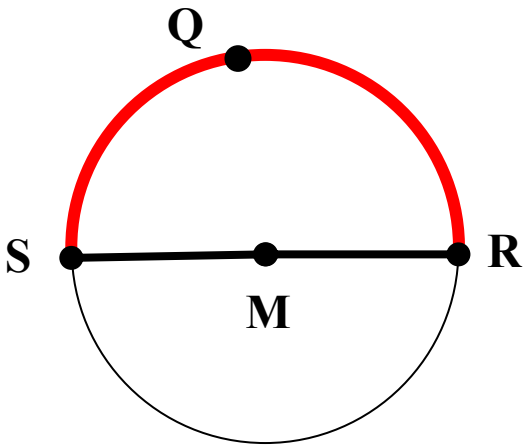
The measure of a major arc is 360° minus the measure of its central angle.

$$\angle PCM = 160^o \rightarrow m\widehat{PUM} = 360^O - 160^o$$

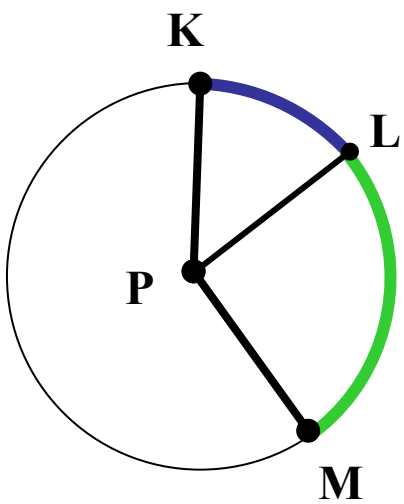
$$m\widehat{PUM} = 200^o$$

The measure of a SEMICIRCLE is 180°

$$m\widehat{SQR} = 360^o$$



Arc Addition Postulate



$$m\widehat{KM} = m\widehat{KL} + m\widehat{LM}$$

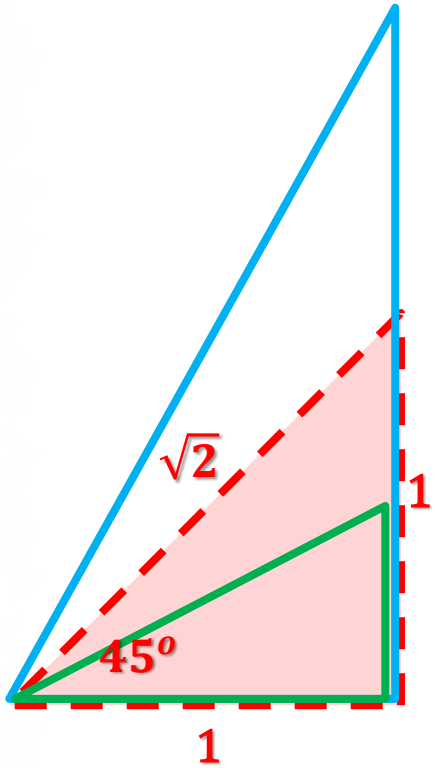
$$\textit{if } \{\textit{Point L is between points K and M in } \widehat{KM}\}$$
$$\textit{then } \{m\widehat{KM} = m\widehat{KL} + m\widehat{LM}\}$$

or

$$\textit{if } \{m\widehat{KM} = m\widehat{KL} + m\widehat{LM}\}$$
$$\textit{then } \{\textit{Point L is between points K and M in } \widehat{KM}\}$$

ANGLE	SINE	COSINE	TANGENT	ANGLE	SINE	COSINE	TANGENT
0	0.0000	1.0000	0.0000	46	0.7193	0.6947	1.0355
1	0.0175	0.9998	0.0175	47	0.7314	0.6820	1.0724
2	0.0349	0.9994	0.0349	48	0.7431	0.6691	1.1106
3	0.0523	0.9986	0.0524	49	0.7547	0.6561	1.1504
4	0.0698	0.9976	0.0699	50	0.7660	0.6428	1.1918
5	0.0872	0.9962	0.0875	51	0.7771	0.6293	1.2349
6	0.1045	0.9945	0.1051	52	0.7880	0.6157	1.2799
7	0.1219	0.9925	0.1228	53	0.7986	0.6018	1.3271
8	0.1392	0.9903	0.1405	54	0.8090	0.5878	1.3764
9	0.1564	0.9877	0.1584	55	0.8192	0.5736	1.4282
10	0.1736	0.9848	0.1763	56	0.8290	0.5592	1.4826
11	0.1908	0.9816	0.1944	57	0.8387	0.5446	1.5399
12	0.2079	0.9781	0.2126	58	0.8480	0.5299	1.6003
13	0.2250	0.9744	0.2309	59	0.8572	0.5150	1.6643
14	0.2419	0.9703	0.2493	60	0.8660	0.5000	1.7321
15	0.2588	0.9659	0.2679	61	0.8746	0.4848	1.8041
16	0.2756	0.9613	0.2867	62	0.8829	0.4695	1.8007
17	0.2924	0.9663	0.3057	63	0.8910	0.4540	1.9626
18	0.3090	0.9511	0.3249	64	0.8988	0.4384	2.0503
19	0.3256	0.9455	0.3443	65	0.9063	0.4226	2.1445
20	0.3420	0.9397	0.3640	66	0.9135	0.4067	2.2461
21	0.3584	0.9336	0.3839	67	0.9205	0.3907	2.3559
22	0.3746	0.9272	0.4040	68	0.9272	0.3746	2.4751
23	0.3907	0.9205	0.4245	69	0.9336	0.3584	2.6051
24	0.4067	0.9135	0.4452	70	0.9397	0.3420	2.7475
25	0.4226	0.9063	0.4663	71	0.9455	0.3256	2.9042
26	0.4384	0.8988	0.4877	72	0.9511	0.3090	3.0777
27	0.4540	0.8910	0.5095	73	0.9563	0.2924	3.2709
28	0.4695	0.8829	0.5317	74	0.9613	0.2756	3.4875
29	0.4848	0.8746	0.5543	75	0.9659	0.2588	3.7321
30	0.5000	0.8660	0.5774	76	0.9703	0.2419	4.0108
31	0.5150	0.8572	0.6009	77	0.9744	0.2249	4.3315
32	0.5299	0.8480	0.6249	78	0.9781	0.2079	4.7047
33	0.5446	0.8387	0.6494	79	0.9816	0.1008	5.1446
34	0.5592	0.8290	0.6745	80	0.9848	0.1736	5.6714
35	0.5736	0.8192	0.7002	81	0.9877	0.1564	6.3139
36	0.5878	0.8000	0.7265	82	0.9903	0.1392	7.1155
37	0.6018	0.7986	0.7536	83	0.9925	0.1219	8.1446
38	0.6157	0.7880	0.7813	84	0.9945	0.1045	9.5147
39	0.6293	0.7771	0.8098	85	0.9962	0.0872	11.4305
40	0.6428	0.7660	0.8391	86	0.9976	0.0698	14.3014
41	0.6561	0.7547	0.8693	87	0.9986	0.0523	19.0824
42	0.6820	0.7311	0.9004	88	0.9994	0.0349	28.6392
44	0.6947	0.7193	0.9657	89	0.9998	0.0174	57.3019
45	0.7071	0.7071	1.0000	90	1.0000	0.0000	*

Trigonometric values from tables first quadrant



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16	0.2756	0.9613	0.2867	62	0.8829	0.4695	1.8807
17	0.2924	0.9563	0.3057	63	0.8910	0.4540	1.9626
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21	0.3584	0.9336	0.3839	67	[illegible]	[illegible]	2.3559
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42	0.6691	0.7431	0.9004	88	0.9994	0.0349	28.6392
43	0.6820	0.7314	0.9325	89	0.9998	0.0174	57.3019
44	0.6947	0.7193	0.9657	90	1.0000	0.0000	*
45	0.7071	0.7071	1.0000				

***Angle to Ratio*:**

$\cos 37° = 0.7986$

$\sin 19° = 0.3256$

$\tan 67° = 2.3559$

ANGLE	SINE	COSINE	TANGENT	ANGLE	SINE	COSINE	TANGENT
0	0.0000	1.0000	0.0000	46	0.7193	0.6947	1.0355
1	0.0175	0.9998	0.0175	47	0.7314	0.6820	1.0724
2	0.0349	0.9994	0.0349	48	0.7431	0.6691	1.1106
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5	0.0872	0.9962	0.0875	51	0.7771	0.6293	1.2349
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22	0.3746	0.9272	0.4040	68	0.9272	0.3746	2.4751
23	[illegible]	0.9205	0.4245	69	0.9336	0.3584	2.6051
24	0.4067	0.9135	0.4452	70	0.9397	0.3420	2.7475
25	0.4226	0.9063	0.4663	71	0.9455	0.3256	2.9042
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27	0.4540	0.8910	0.5095	73	0.9563	0.2924	3.2709
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30	0.5000	0.8660	0.5774	76	0.9703	0.2419	4.0108
31	0.5150	0.8572	0.6009	77	0.9744	0.2249	4.3315
32	0.5299	0.8480	0.6249	78	0.9781	0.2079	4.7047
33	0.5446	0.8387	0.6494	79	0.9816	0.1908	5.1446
34	0.5592	0.8290	0.6745	80	0.9848	0.1736	5.6714
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37	0.6018	0.7986	0.7536	83	0.9925	0.1219	8.1446
38	0.6157	0.7880	0.7813	84	[illegible]	[illegible]	9.5147
39	0.6293	0.7771	0.8098	85	0.9962	0.0872	11.4305
40	0.6428	0.7660	0.8391	86	0.9976	0.0698	14.3014
41	0.6561	0.7547	0.8693	87	0.9986	0.0523	19.0824
42	0.6691	0.7431	0.9004	88	0.9994	0.0349	28.6392
43	0.6820	0.7314	0.9325	89	0.9998	0.0174	57.3019
44	0.6947	0.7193	0.9657	90	1.0000	0.0000	*
45	0.7071	0.7071	1.0000				

Ratio to Angle:

$\cos^{-1}(0.9205) = 23°$

$\sin^{-1}(0.0872) = 5°$

$\tan^{-1}(9.5144) = 84°$

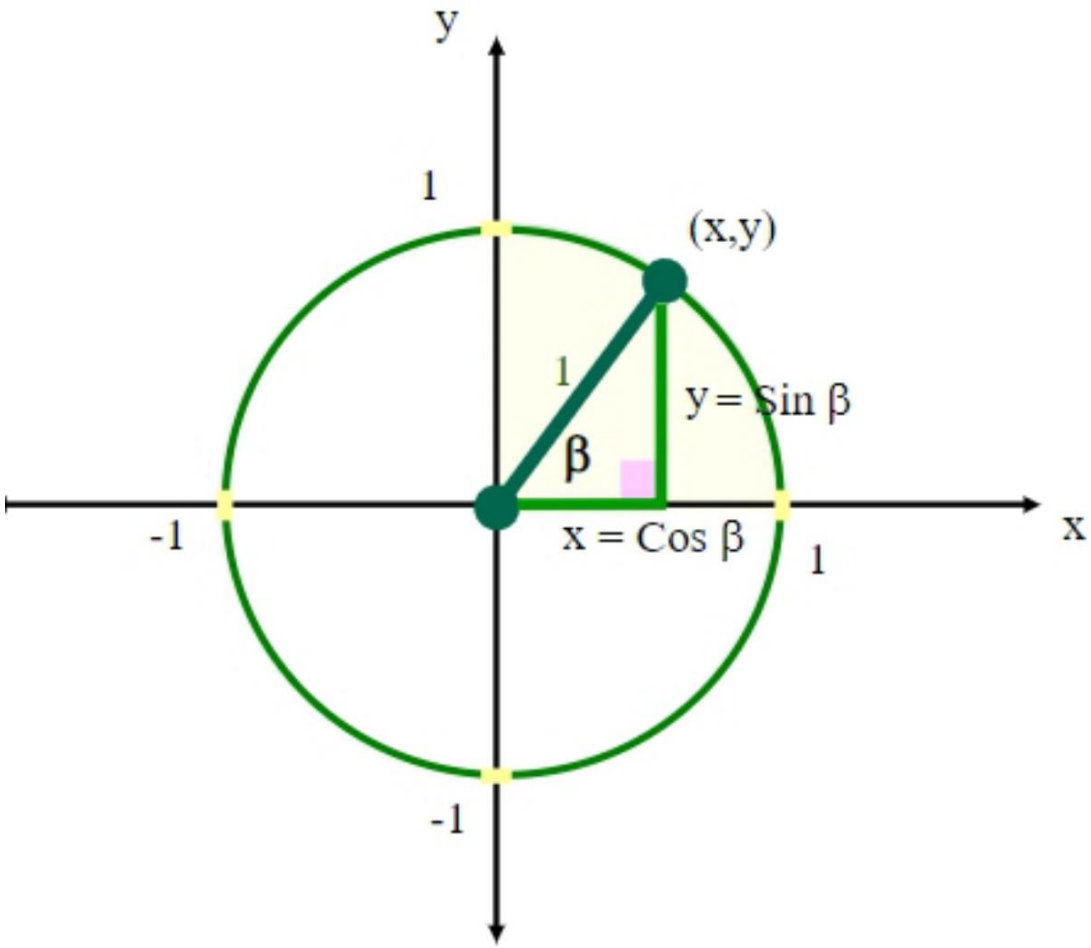
Ratio to Angle:

$\text{acos}(0.9205) = 23°$

$\text{asin}(0.0872) = 5°$

$\text{atan}(9.5144) = 84°$

TRIGONOMETRIC CIRCLE: First Quadrant



$$sine = \frac{opposite\ side}{hypotenuse}$$

$$cosine = \frac{adjacent\ side}{hypotenuse}$$

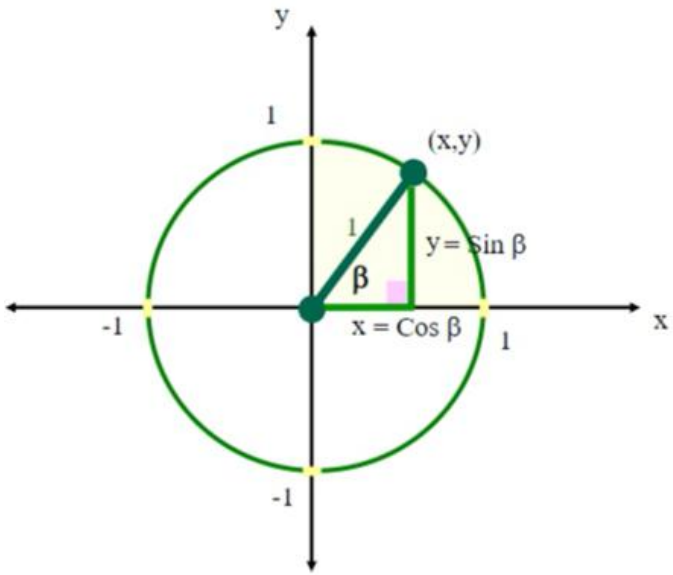
$$tangent = \frac{opposite\ side}{adjacent\ side}$$

$$cotangent = \frac{adjacent\ side}{opposite\ side}$$

$$secant = \frac{hypotenuse}{adjacent\ side}$$

$$cosecant = \frac{hypotenuse}{opposite\ side}$$

Trigonometric Circle



$$\text{Sin } \beta = \frac{y}{1}$$

$$\text{Sin } \beta = y$$

$$\text{Cos } \beta = \frac{x}{1}$$

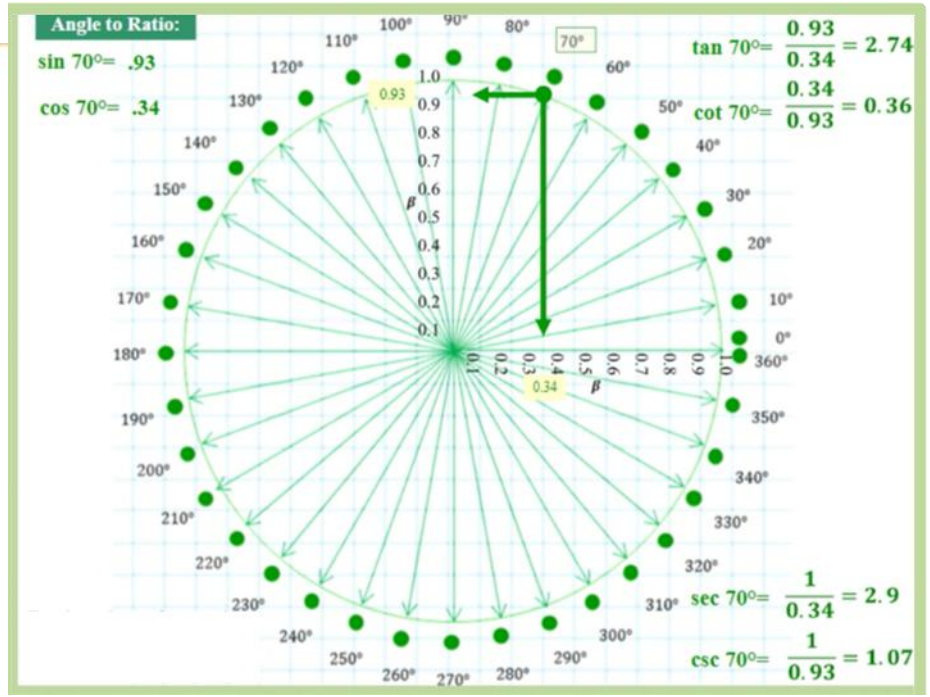
$$\text{Cos } \beta = x$$

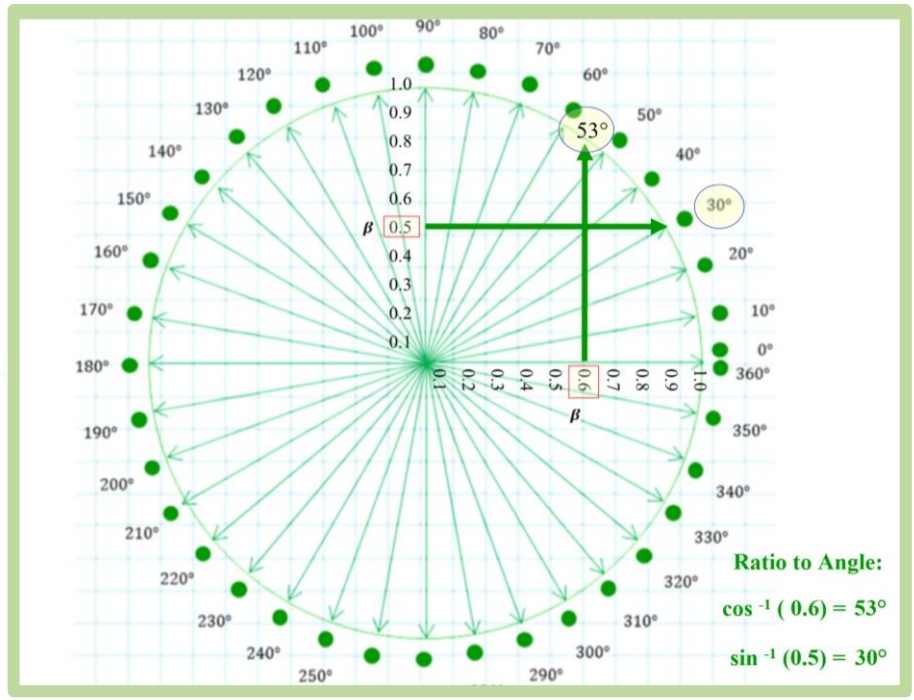
$$\text{Tan } \beta = \frac{y}{x}$$

$$\text{Cot } \beta = \frac{x}{y}$$

$$\text{Sec } \beta = \frac{1}{x}$$

$$\text{Csc } \beta = \frac{1}{y}$$

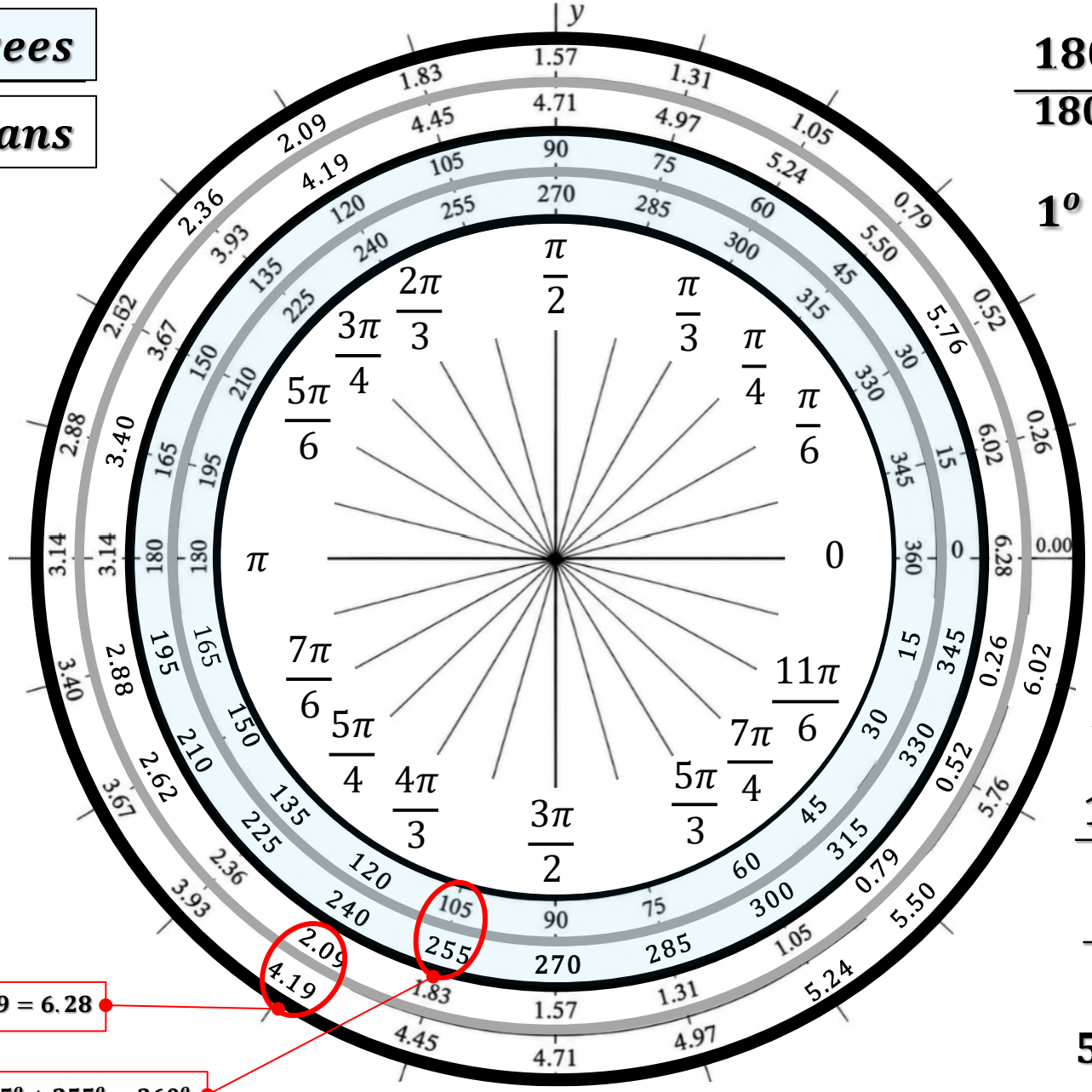




Using the first quadrant to get all 6 trigonometric ratios

Degrees

Radians



$$\frac{180^o}{180^o} = \frac{\pi}{180^o}$$

$$1^o = \frac{3.14}{180} \approx 0.018$$

Radians

Radians

$$180^o = \pi$$

$$\frac{180^o}{\pi} = \frac{\pi}{\pi}$$

$$\frac{180^o}{3.14} = \frac{\pi}{\pi}$$

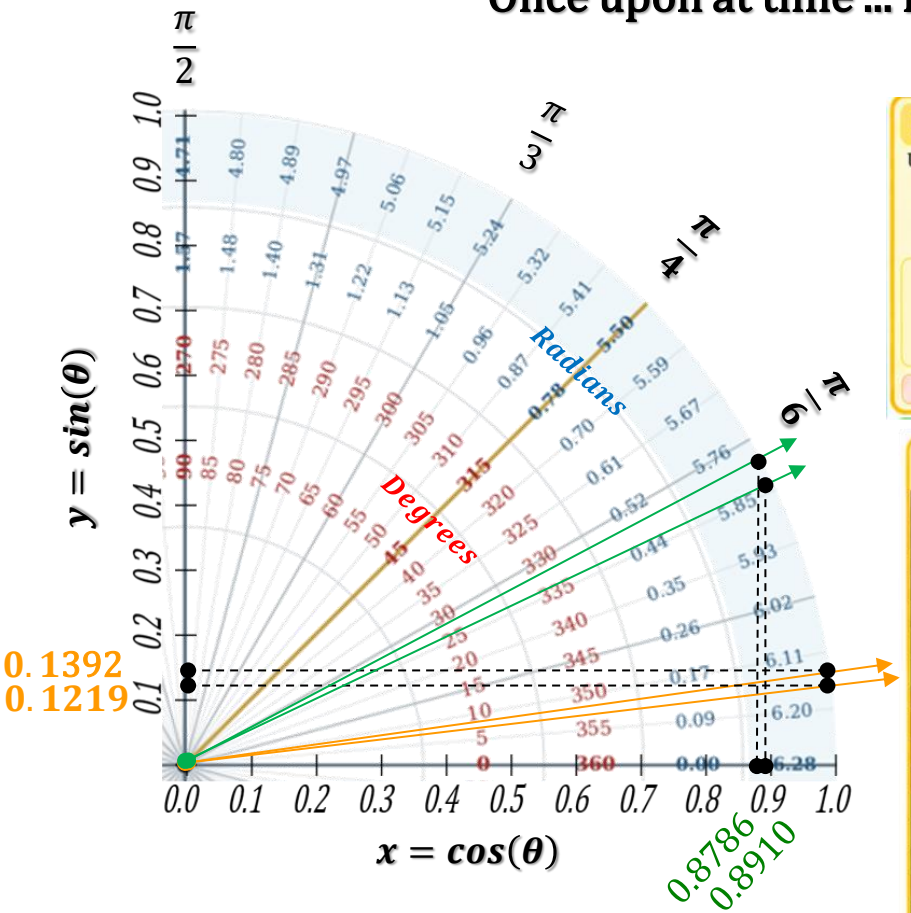
$$57.3^o = 1\ Radian$$

$2.09 + 4.19 = 6.28$

$105^o + 255^o = 360^o$

Note: $\pi \approx 3.14$ *including more decimals more precision*

Once upon at time ... no calculators were available.



Example 1: Estimate $\sin(7.5°)$

Use the table values $\sin(7°) = 0.1219$ and $\sin(8°) = 0.1392$.

0.1219 — x — 0.1392

7° — 7.5° — 8°

$$\frac{(7.5-7)}{(8-7)} = 0.5$$

$$\sin(7.5°) \approx 0.1219 + 0.5(0.1392 - 0.1219)$$
$$\approx 0.1219 + 0.5(0.0173)$$
$$\approx 0.13055 \approx \boxed{0.1306}$$

This is an estimate between 7° and 8°.

Example 2: Estimate $\cos(28°)$

Use the table values $\cos(27°) = 0.8910$ and $\cos(29°) = 0.8746$.

0.8910 — x — 0.8746

27° — 28° — 29°

$$\frac{(28-27)}{(29-27)} = \frac{1}{2} = 0.5$$

$$\cos(28°) \approx 0.8910 + 0.5(0.8746 - 0.8910)$$
$$\approx 0.8910 + 0.5(-0.0164)$$
$$\approx \boxed{0.8828}$$

The exact value is close, so a small interpolation error is normal.

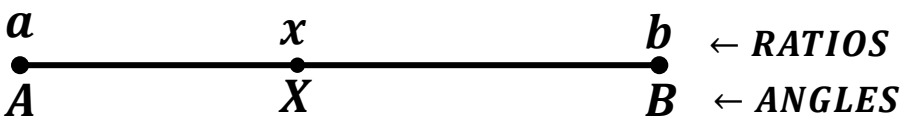
Table 1: 0° to 15° (by 1°)

ANGLE	SINE	COSINE
0	0.0000	1.0000
1	0.0175	0.9998
2	0.0349	0.9994
3	0.0523	0.9986
4	0.0698	0.9976
5	0.0872	0.9962
6	0.1045	0.9945
7	0.1219	0.9925
8	0.1392	0.9903
9	0.1564	0.9877
10	0.1736	0.9848
11	0.1908	0.9816
12	0.2079	0.9781
13	0.2250	0.9744
14	0.2419	0.9703
15	0.2588	0.9659

Table 2: 20° to 35° (by 1°)

ANGLE	SINE	COSINE
20	0.3420	0.9397
21	0.3584	0.9336
22	0.3746	0.9272
23	0.3907	0.9205
24	0.4067	0.9135
25	0.4226	0.9063
26	0.4384	0.8988
27	0.4540	0.8910
28	0.4695	0.8829
29	0.4848	0.8746
30	0.5000	0.8660
31	0.5150	0.8572
32	0.5299	0.8480
33	0.5446	0.8387
34	0.5592	0.8290
35	0.5736	0.8192

Many times a trigonometric value when retrieved from tables can not be taken directly from the table and makes necessary to use what we know from proportional reasoning to set up a proportion and perform an **interpolation**

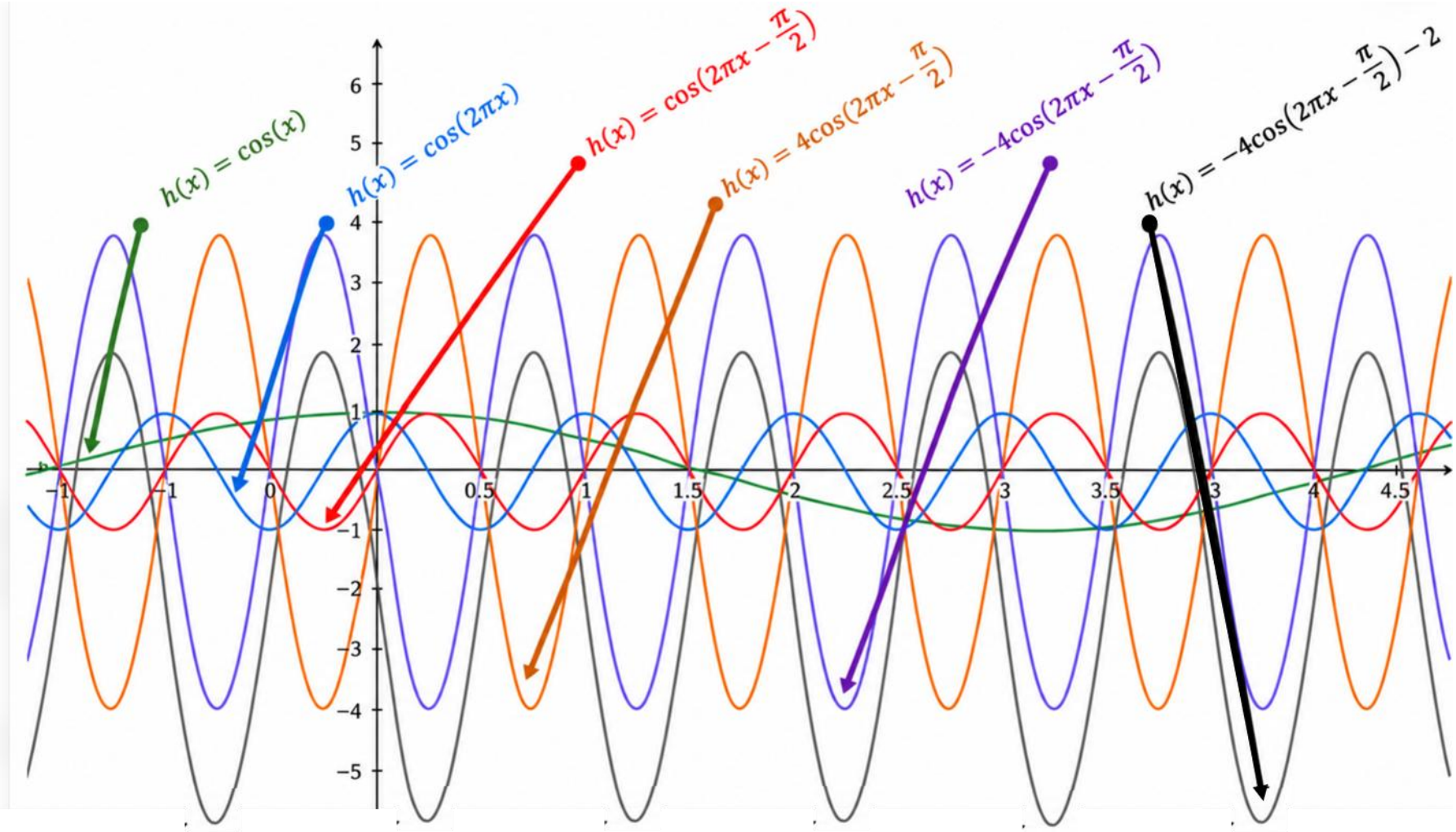


$$\frac{x-a}{X-A} = \frac{b-a}{B-A} \rightarrow x = a + (X-A)\left(\frac{b-a}{B-A}\right)$$

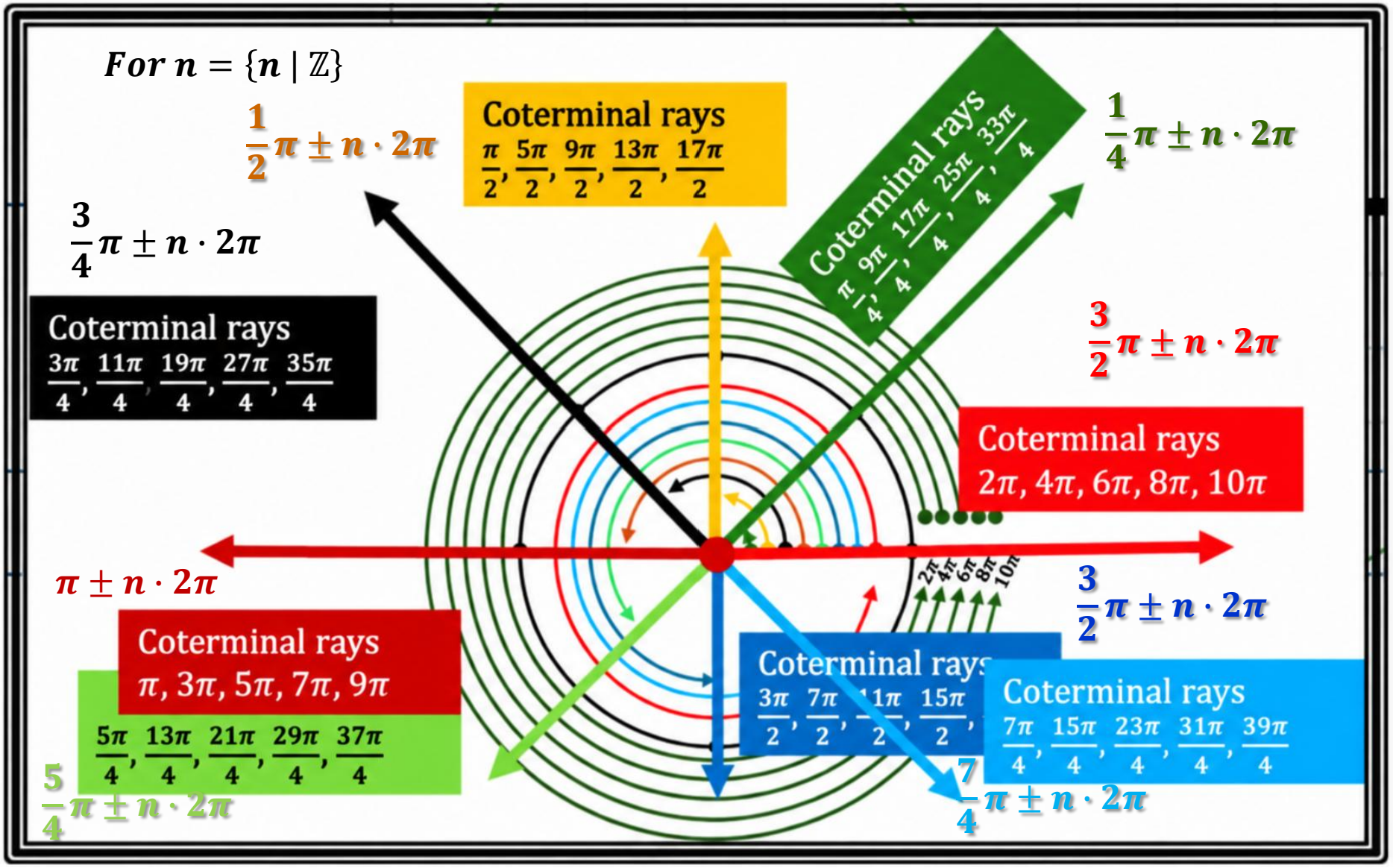
Common Trigonometric Values for Sine and Cosine

θ		QUADRANT I				θ		QUADRANT II				θ		QUADRANT III				θ		QUADRANT VI			
RADIANS	DEGREES	*Sin*		*Cos*		RADIANS	DEGREES	*Sin*		*Cos*		RADIANS	DEGREES	*Sin*		*Cos*		RADIANS	DEGREES	*Sin*		*Cos*	
$\frac{\pi}{12}$	15°		.26		.97	$\frac{7}{12}\pi$	105°		.97		−.26	$\frac{13}{12}\pi$	195°		−.26		−.97	$\frac{19}{12}\pi$	285°		−.97		.26
$\frac{\pi}{6}$	30°	$\frac{1}{2}$	.5	$\frac{\sqrt{3}}{2}$	.87	$\frac{2}{3}\pi$	120°	$\frac{\sqrt{3}}{2}$	.87	$-\frac{1}{2}$	−.5	$\frac{7}{6}\pi$	210°	$-\frac{1}{2}$	−.5	$-\frac{\sqrt{3}}{2}$	−.87	$\frac{5}{3}\pi$	300°	$-\frac{\sqrt{3}}{2}$	−.87	$\frac{1}{2}$	.5
$\frac{\pi}{4}$	45°	$\frac{\sqrt{2}}{2}$	.71	$\frac{\sqrt{2}}{2}$	.71	$\frac{3}{4}\pi$	135°	$\frac{\sqrt{2}}{2}$	.71	$-\frac{\sqrt{2}}{2}$	−.71	$\frac{5}{4}\pi$	225°	$-\frac{\sqrt{2}}{2}$	−.71	$-\frac{\sqrt{2}}{2}$	−.71	$\frac{7}{4}\pi$	315°	$-\frac{\sqrt{2}}{2}$	−.71	$\frac{\sqrt{2}}{2}$	.71
$\frac{\pi}{3}$	60°	$\frac{\sqrt{3}}{2}$	.87	$\frac{1}{2}$	.5	$\frac{5}{6}\pi$	150°	$\frac{1}{2}$	.5	$-\frac{\sqrt{3}}{2}$	−.87	$\frac{4}{3}\pi$	240°	$-\frac{\sqrt{3}}{2}$	−.87	$-\frac{1}{2}$	−.5	$\frac{11}{6}\pi$	330°	$-\frac{1}{2}$	−.5	$\frac{\sqrt{3}}{2}$	.87
$\frac{5}{12}\pi$	75°		.97		.26	$\frac{11}{12}\pi$	165°		.26		−.97	$\frac{17}{12}\pi$	255°		−.97		−.26	$\frac{13}{12}\pi$	345°		−.26		.97
$\frac{\pi}{2}$	90°		1	0		π	180°	0			−1	$\frac{3}{2}\pi$	270°		−1		0	2π	360°		0		1

$\boldsymbol{Effects\ of\ parameters\ in\ a\ trigonometric\ function}$:
$\boldsymbol{f(x) = A \cdot cos(B \cdot x + C) + D}$



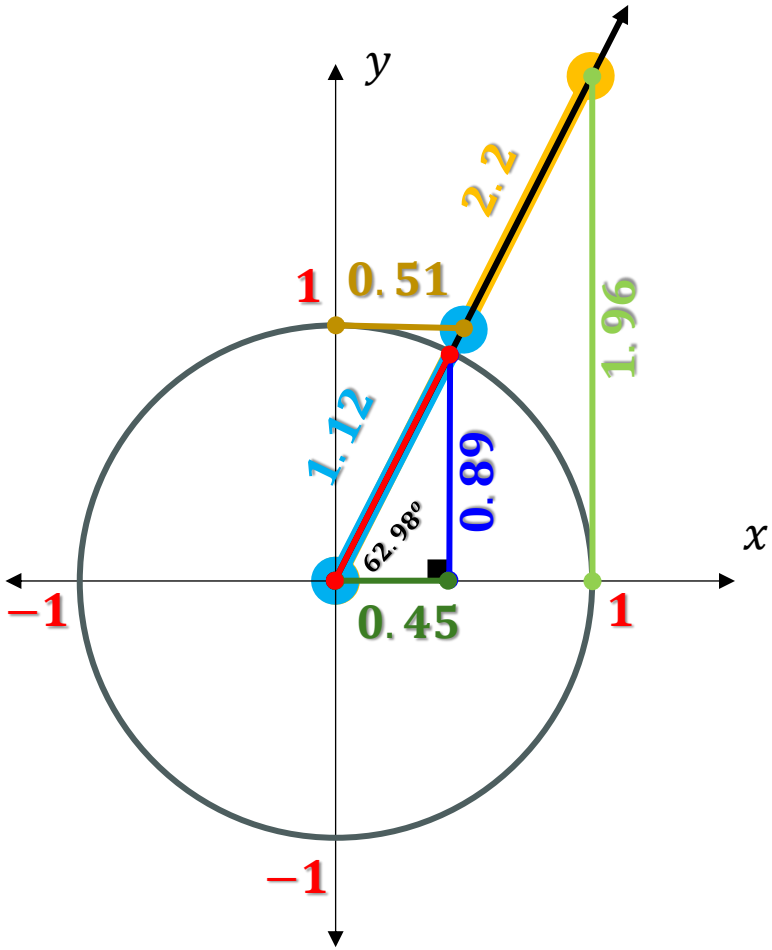
Terminal Rays and Periodic Concept

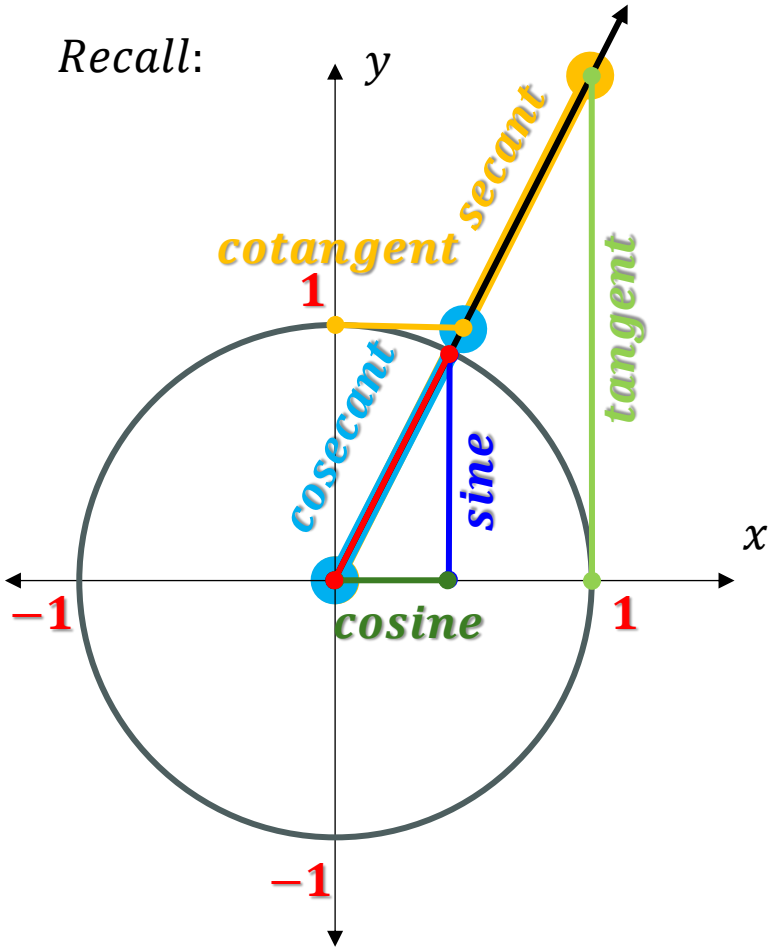


UNIT CIRCLE:

$\sin(62.98^o) = 0.89$ $\cos(62.98^o) = 0.45$ $\tan(62.98^o) = 1.96$

$\cot(62.98^o) = 0.51$ $\sec(62.98^o) = 2.2$ $\csc(62.98^o) = 1.12$

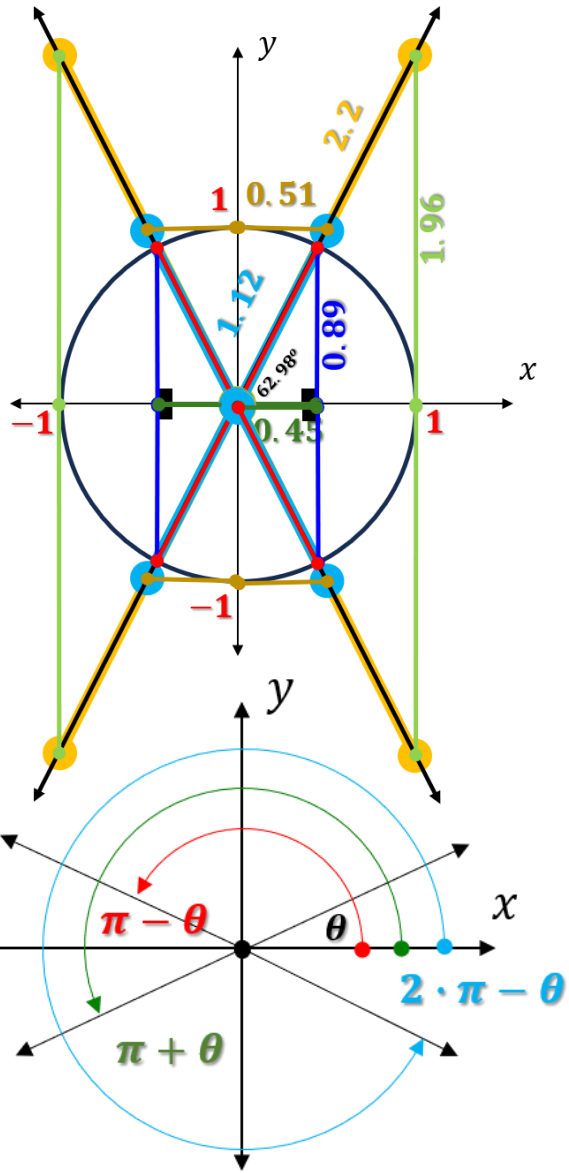




This graphic allows to connect rotations and reflections plus sign implications

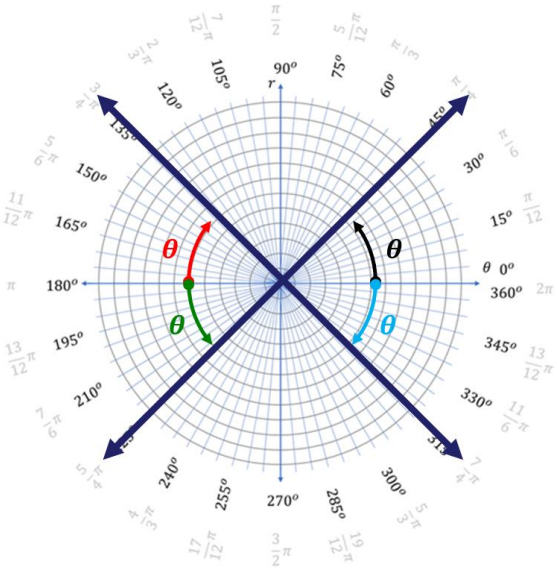
$$\sin = \frac{+}{+} = +$$
$$\cos = \frac{-}{+} = -$$
$$\tan = \frac{+}{-} = -$$
$$\cot = \frac{-}{+} = -$$
$$\sec = \frac{-}{+} = -$$
$$\csc = \frac{+}{+} = +$$

$$\sin = \frac{-}{+} = -$$
$$\cos = \frac{-}{+} = -$$
$$\tan = \frac{-}{-} = +$$
$$\cot = \frac{-}{-} = +$$
$$\sec = \frac{+}{-} = -$$
$$\csc = \frac{+}{-} = -$$



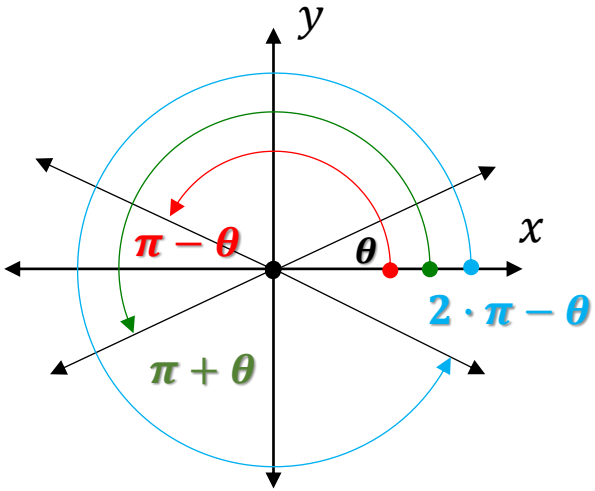
$$\sin = \frac{+}{+} = +$$
$$\cos = \frac{+}{+} = +$$
$$\tan = \frac{+}{+} = +$$
$$\cot = \frac{+}{+} = +$$
$$\sec = \frac{+}{+} = +$$
$$\csc = \frac{+}{+} = +$$

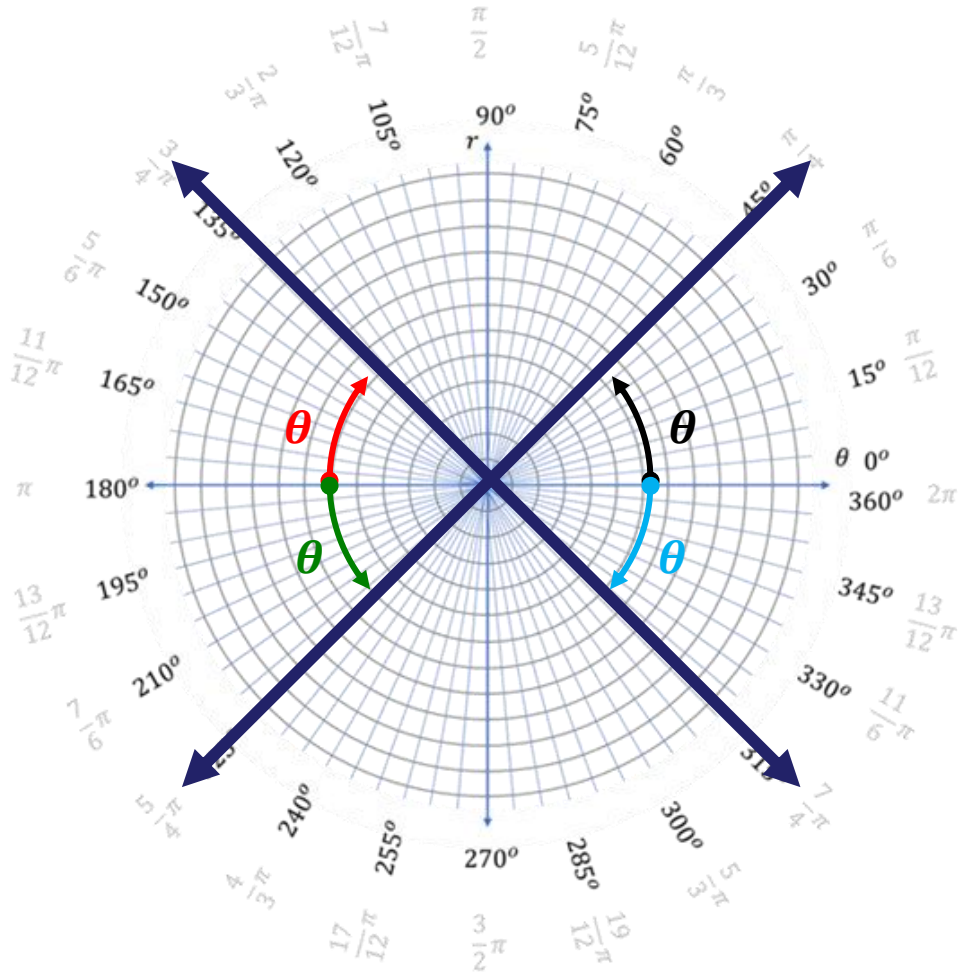
$$\sin = \frac{-}{+} = -$$
$$\cos = \frac{+}{+} = +$$
$$\tan = \frac{-}{+} = -$$
$$\cot = \frac{+}{-} = -$$
$$\sec = \frac{+}{+} = +$$
$$\csc = \frac{+}{-} = -$$



Recall:
The angles in Standard Position are as here.
It is how to take them in the calculations

Expressing Angles in in terms of π and θ





Reciprocal Trigonometric Ratios:

$$\sin\theta = \frac{1}{\csc\theta}$$

$$\cos\theta = \frac{1}{\sec\theta}$$

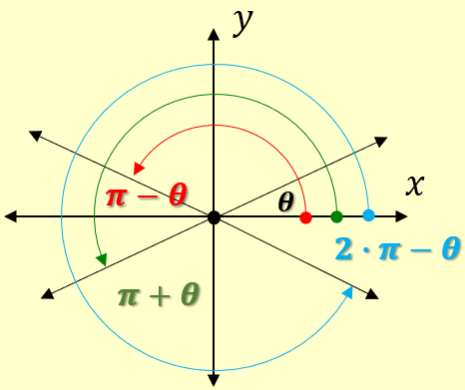
$$\tan\theta = \frac{1}{\cot\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\csc\theta = \frac{1}{\sin\theta}$$

Expressing Angles in Terms of π and θ



Reflecting about $x = 0$

$$\sin(\pi - \theta) = \sin\theta$$

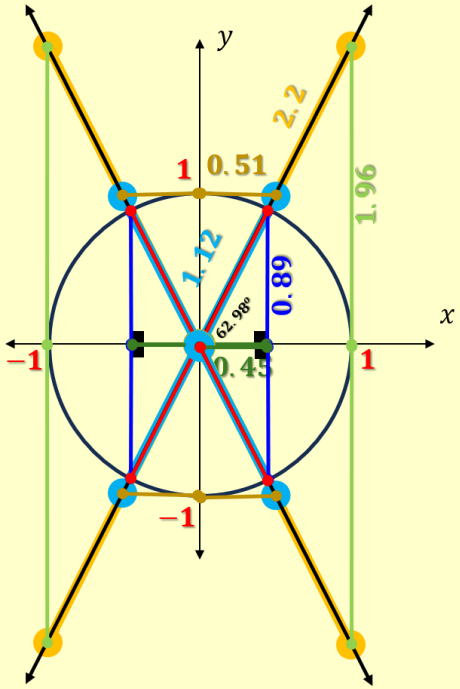
$$\cos(\pi - \theta) = -\cos\theta$$

$$\tan(\pi - \theta) = -\tan\theta$$

$$\cot(\pi - \theta) = -\cot\theta$$

$$\sec(\pi - \theta) = -\sec\theta$$

$$\sec(\pi - \theta) = \csc\theta$$



Reflecting about y=0

$$\sin(-\theta) = -\sin\theta$$

$$\cos(-\theta) = \cos\theta$$

$$\tan(-\theta) = -\tan\theta$$

$$\cot(-\theta) = -\cot\theta$$

$$\sec(-\theta) = \sec\theta$$

$$\sec(-\theta) = -\csc\theta$$

Cofunctions Trigonometric Ratios:

$$\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$$

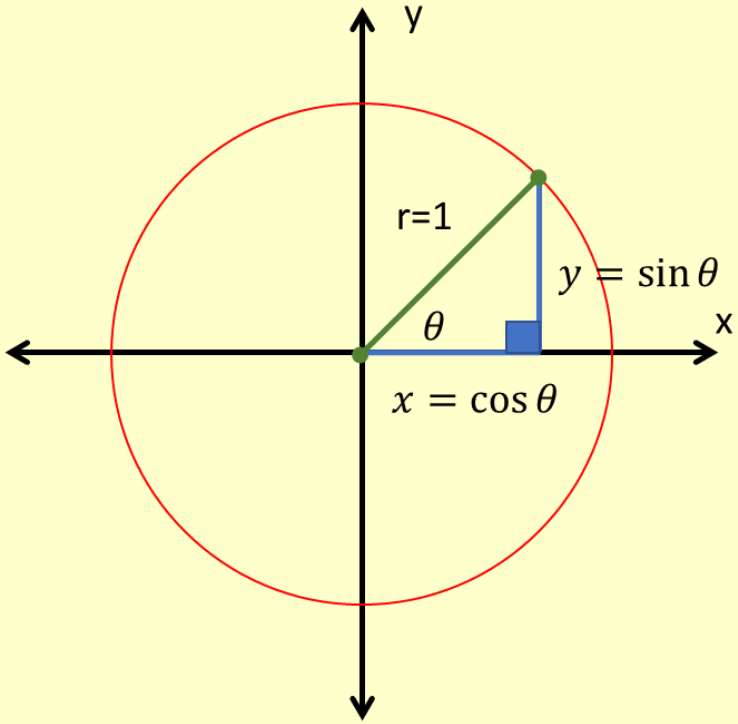
$$\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$$

$$\tan\theta = \cot\left(\frac{\pi}{2} - \theta\right)$$

$$\cot\theta = \tan\left(\frac{\pi}{2} - \theta\right)$$

$$\sec\theta = \csc\left(\frac{\pi}{2} - \theta\right)$$

$$\csc\theta = \sec\left(\frac{\pi}{2} - \theta\right)$$



Pythagorean Theorem
(Proof for Pythagorean Identities):

$$r^2 = x^2 + y^2 \Rightarrow \boxed{1 = sin^2\theta + cos^2\theta}$$

$$\frac{1}{cos^2\theta} = \frac{sin^2\theta}{cos^2\theta} + \frac{cos^2\theta}{cos^2\theta}$$

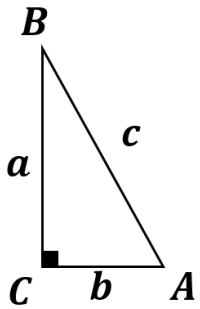
$$\boxed{sec^2\theta = tan^2\theta + 1}$$

$$\frac{1}{sin^2\theta} = \frac{sin^2\theta}{sin^2\theta} + \frac{cos^2\theta}{sin^2\theta}$$

$$\boxed{csc^2\theta = 1 + cot^2\theta}$$

Distance Formula between two points in a plane:

Recall Pythagorean Theorem:



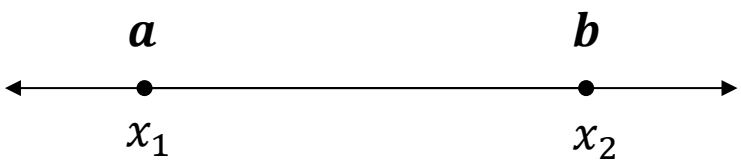
$$c^2 = a^2 + b^2$$

$$|c| = \sqrt{a^2 + b^2}$$

$$c = \pm\sqrt{a^2 + b^2}$$

$(-)$ *no sense here*

and Recall:

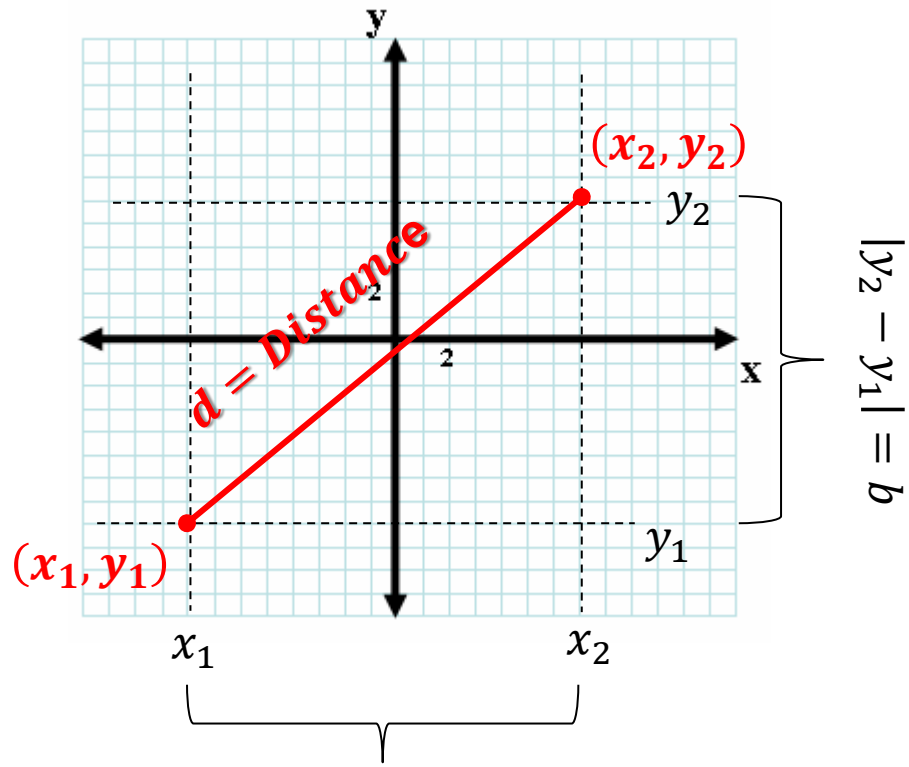


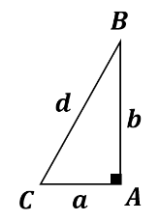
$$Distance_{a\ to\ b} = |b - a|$$

$$|b - a| = \begin{cases} -(b-a), & (b-a) < 0 \\ (b-a), & (b-a) \geq 0 \end{cases}$$

and

$$\sqrt{x^2} = |x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$$





Combining ideas at the left:

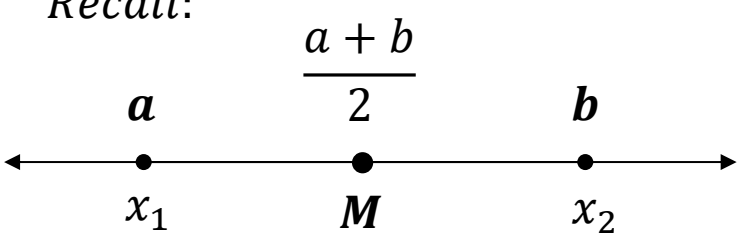
Distance between 2 points

(x_2, y_2) ***and*** (x_1, y_1) *in the* ***coordinate plane***:

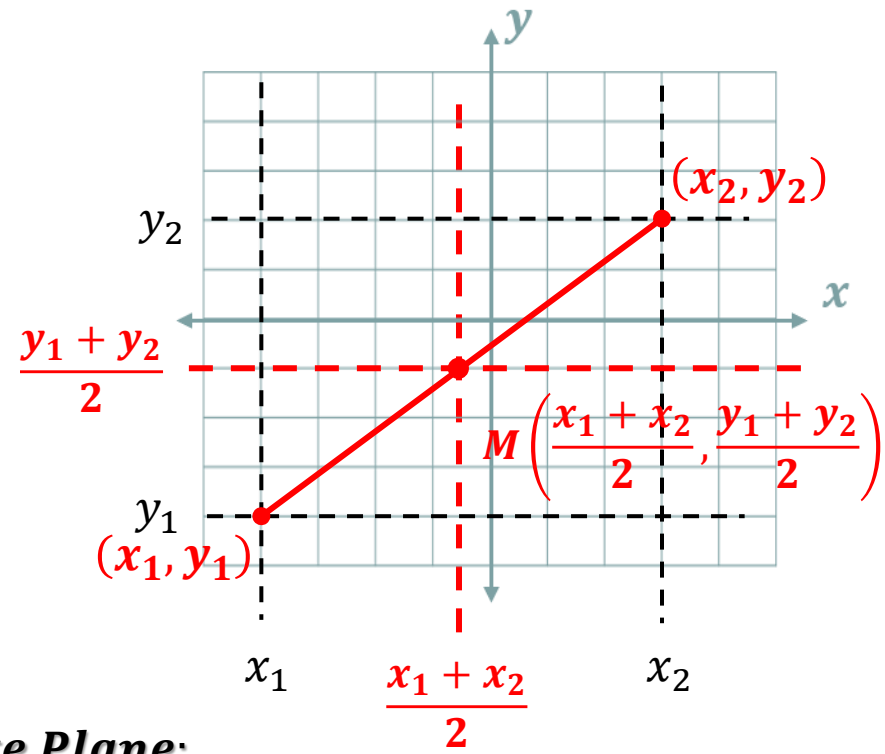
$$\boldsymbol{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

Midpoint Formula between two points in a plane:

Recall:



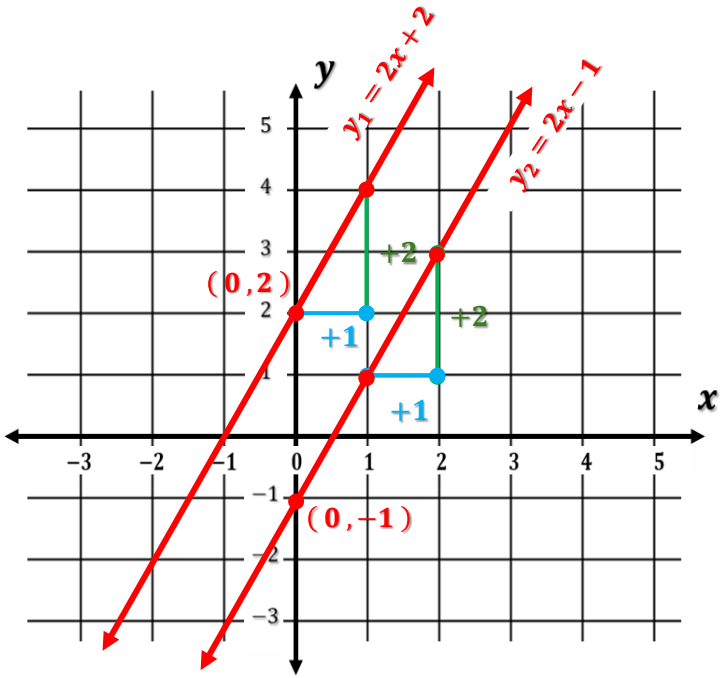
$$Midpoint_{\ a\ to\ b} = \frac{a+b}{2}$$



Midpoint Formula in the Coordinate Plane:

$$M(x, y) = \left(\frac{\boldsymbol{x_1 + x_2}}{\boldsymbol{2}}, \frac{\boldsymbol{y_1 + y_2}}{\boldsymbol{2}}\right)$$

Parallel Lines



$$y_1 = m \cdot x + b \rightarrow Slope - Intercept\ Form$$

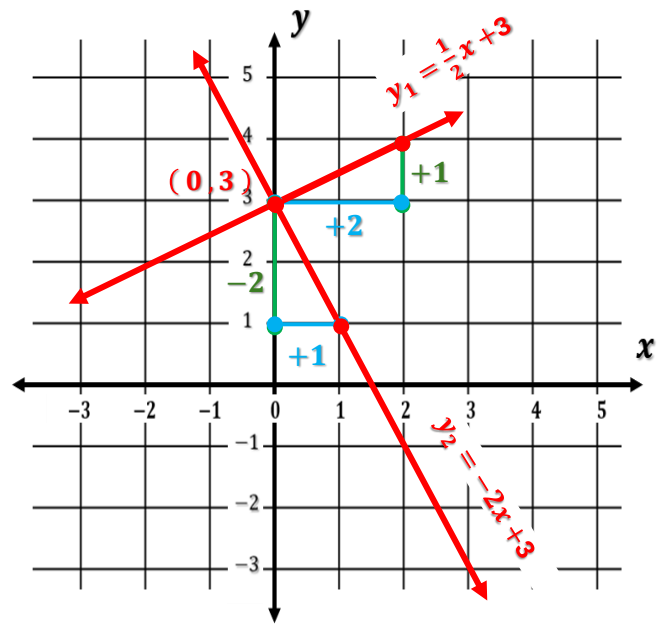
$$m = \frac{rise}{run} \rightarrow Slope \quad \& \quad (0, b) \rightarrow y - intercept$$

$$y_1 = \frac{+2}{+1}x + 2 = 2x + 2 \quad \rightarrow \quad m_1 = 2$$

$$y_2 = \frac{+2}{+1}x - 1 = 2x - 1 \quad \rightarrow \quad m_2 = 2$$

$$m_1 = m_2 = 2 \quad \Longleftrightarrow \quad Lines\ are\ \parallel' s$$

Perpendicular Lines



$$y_1 = m \cdot x + b \rightarrow Slope - Intercept\ Form$$

$$m = \frac{rise}{run} \rightarrow Slope \quad \& \quad (0, b) \rightarrow y - intercept$$

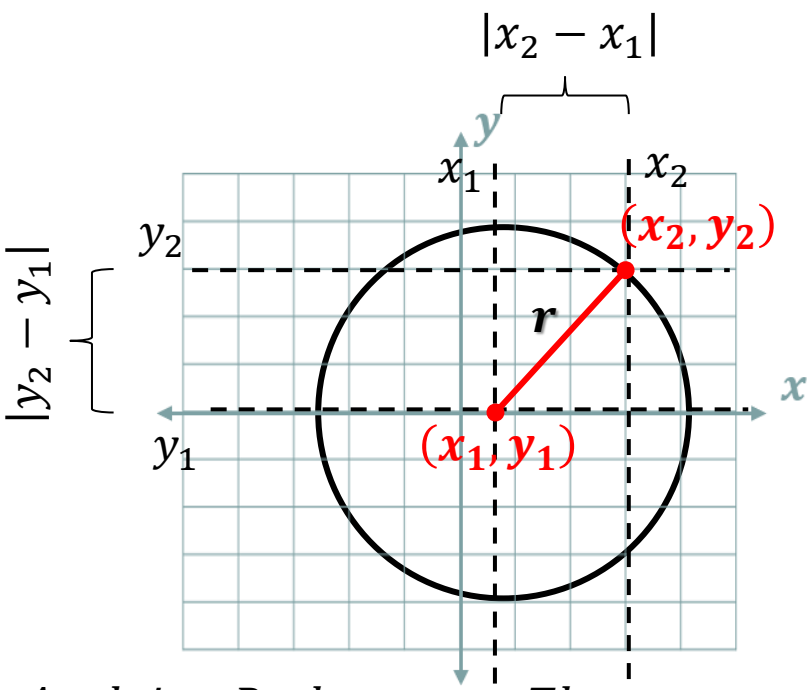
$$y_1 = \frac{+1}{+2}x + 3 = \frac{1}{2}x + 3 \quad \rightarrow \quad m_1 = \frac{1}{2}$$

$$y_2 = \frac{-2}{+1}x + 3 = -2x + 3 \quad \rightarrow \quad m_2 = -2$$

$$m_1 \cdot m_2 = \left(\frac{1}{2}\right) \cdot (-2) = \left(\frac{+1}{+2}\right) \cdot \left(\frac{-2}{+1}\right) = \frac{-2}{+2} = -1$$

$$Lines\ are\ \perp' s \quad \Longleftrightarrow \quad m_1 \cdot m_2 = -1 \rightarrow \begin{cases} m_1 = -\dfrac{1}{m_2} \\ m_2 = -\dfrac{1}{m_1} \end{cases}$$

Conics: Circle Formula



Applying Pythagorean Theorem

$$(x_2 - x_1)^2 + (y_2 - y_1)^2 = r^2$$

where:

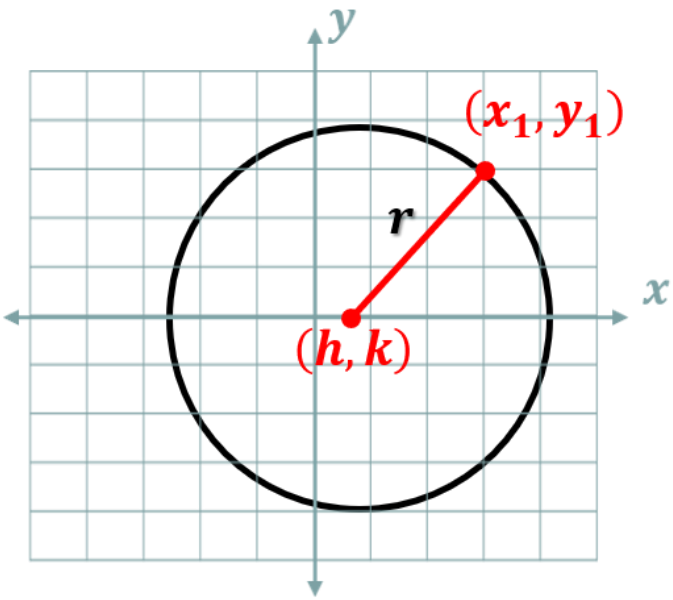
(x_1, y_1) → ***center of circle***

(x_2, y_2) → ***point on the circumference***

r → *radius*

or

$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



Circle Formula:

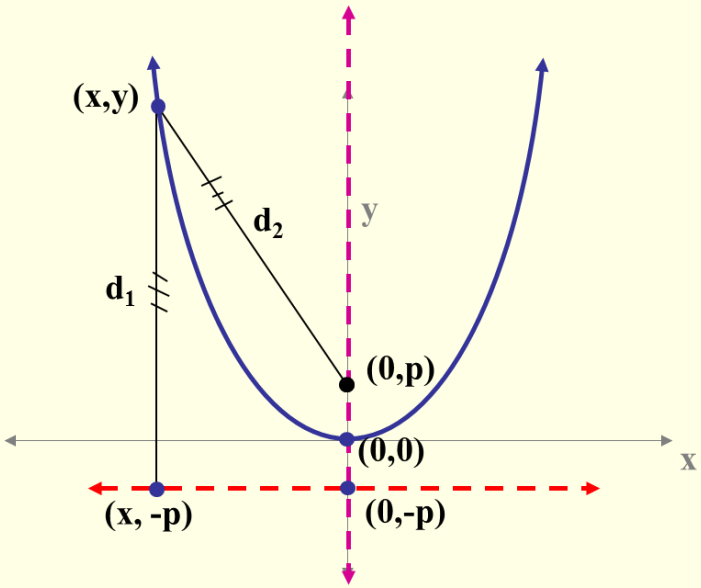
$$(x_1 - h)^2 + (y_1 - k)^2 = r^2$$

(h, k) → ***center of circle***

(x_1, y_1) → ***point on circumference***

$$r = \sqrt{(x_1 - h)^2 + (y_1 - k)^2}$$

Solving for y:



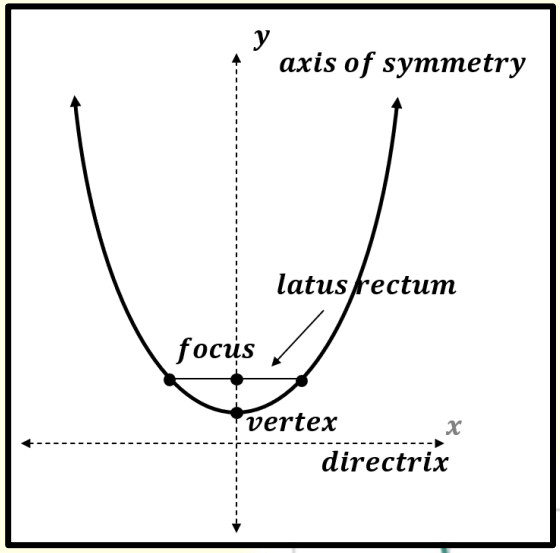
$$\boldsymbol{d_1 = d_2}$$

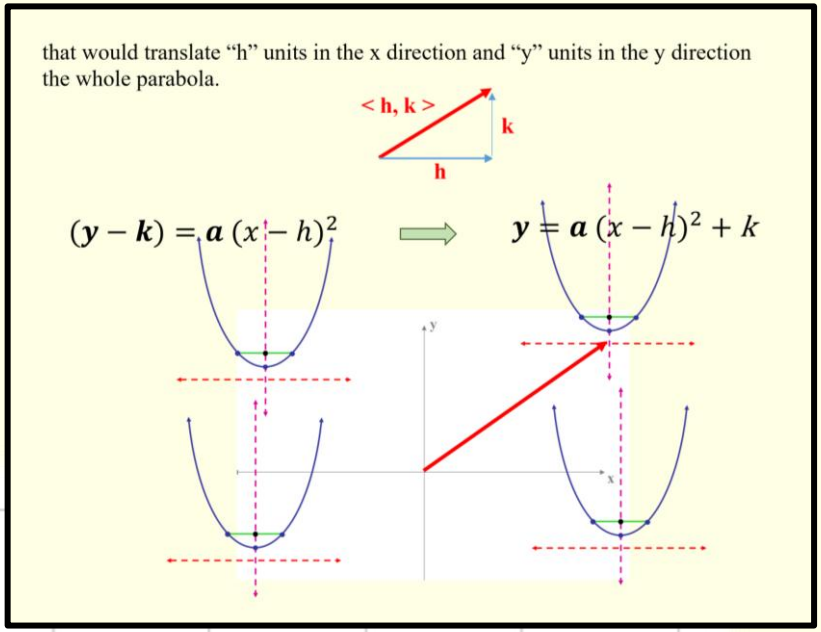
$$\sqrt{(x - x)^2 + (y - (-p))^2} = \sqrt{(x - 0)^2 + (y - p)^2}$$

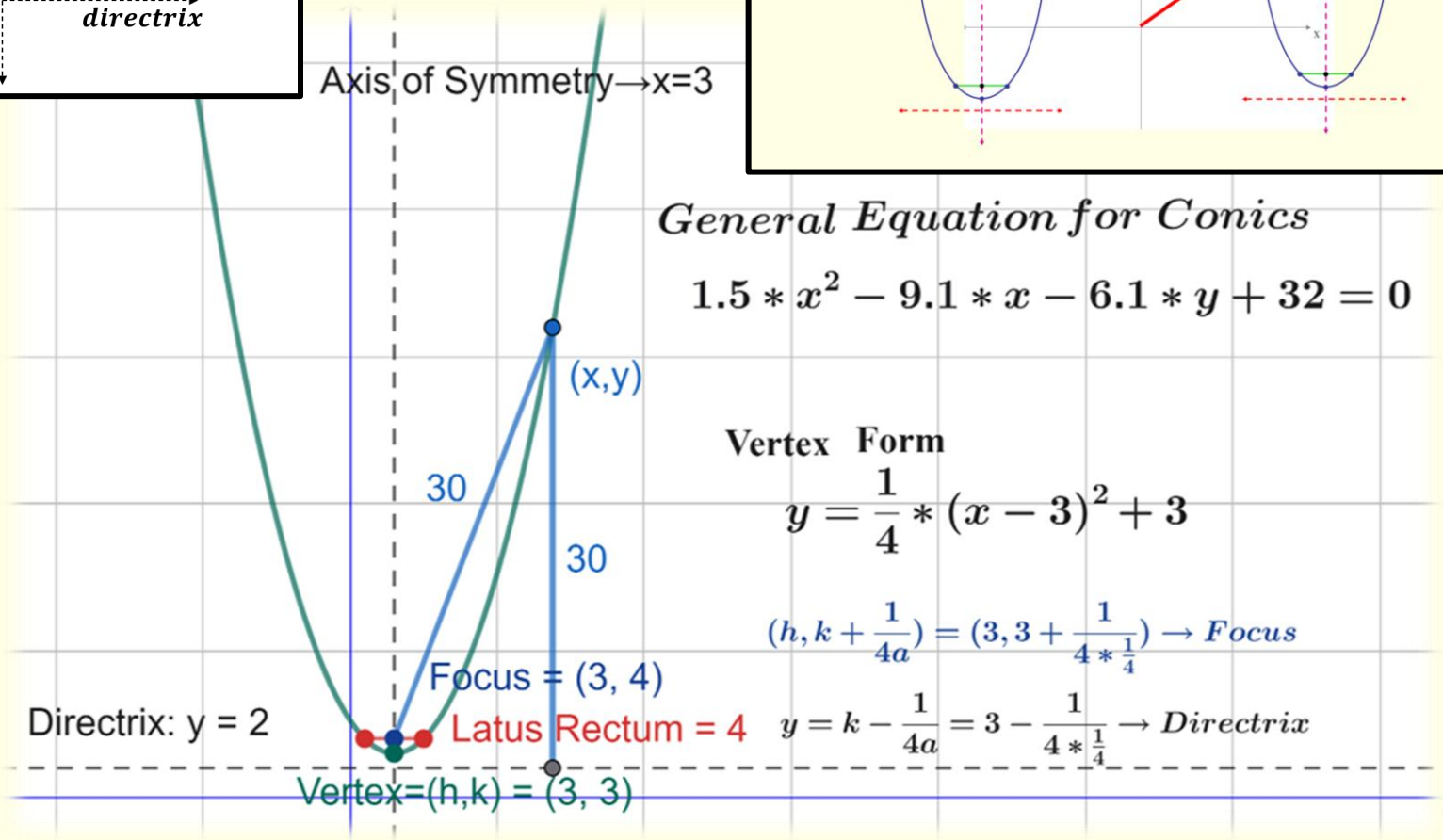
$$\boldsymbol{y} = \frac{1}{4p} x^2$$ Replacing $\frac{1}{4p}$ by "a"

$$\boldsymbol{y = a}\, x^2$$

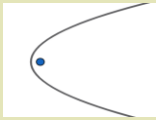
Now the parabola may be in a different location that centered at the origin. To accomplish this we would need to use a **translation vector < h, k >** that would translate "h" units in the x direction and "y" units in the y direction the whole parabola.


y
axis of symmetry
latus rectum
focus
vertex
x
directrix


that would translate "h" units in the x direction and "y" units in the y direction the whole parabola.
< h, k >
k
h
$(y-k) = a\,(x-h)^2$
$y = a\,(x-h)^2 + k$


Axis of Symmetry→x=3
(x,y)
30
30
Focus = (3, 4)
Directrix: y = 2
Latus Rectum = 4
Vertex=(h,k) = (3, 3)
General Equation for Conics
$1.5 * x^2 - 9.1 * x - 6.1 * y + 32 = 0$
Vertex Form
$y = \frac{1}{4} * (x-3)^2 + 3$
$(h, k + \frac{1}{4a}) = (3, 3 + \frac{1}{4 * \frac{1}{4}}) \rightarrow Focus$
$y = k - \frac{1}{4a} = 3 - \frac{1}{4 * \frac{1}{4}} \rightarrow Directrix$

PARABOLAS: SUMMARY



Form of equation	$y = a(x-h)^2 + k$	$x = a(y-k)^2 + h$
Axis of symmetry	$x = h$	$y = k$
Vertex	(h, k)	(h, k)
Focus	$\left(h, k + \frac{1}{4 \cdot a}\right)$	$\left(h + \frac{1}{4 \cdot a}, k\right)$
Directrix	$y = k - \frac{1}{4 \cdot a}$	$x = h - \frac{1}{4 \cdot a}$
Direction of openning	***Upward if*** $a > 0$ ***Downward if*** $a < 0$	***Right if*** $a > 0$ ***Left if*** $a < 0$
Length of latus rectum	$\left\lvert \frac{1}{4 \cdot a} \right\rvert$ units	$\left\lvert \frac{1}{4 \cdot a} \right\rvert$ units